Selection and Sorting of Heterogeneous Firms Through Competitive Pressures

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Teaching Slides

Introduction

Competitive Pressures on Heterogeneous Firms

Main Questions: How do more ***competitive pressures***, due to entry of new firms, caused by lower ***entry cost*** or larger ***market size***, affect firms with different productivity?

- Selection of firms
- Distribution of firm size (in revenue, profit and employment), Distribution of markup and pass-through rates, etc.
- Sorting of firms across markets with different market sizes

Existing Monopolistic Competition Models with Heterogenous Firms

- Melitz (2003): under **CES** Demand System (DS)
  - MC firms sell their products at an exogenous & common markup rate, *unresponsive to competitive pressures*
  - Market size: no effect on distribution of firm types nor their behaviors; All adjustments at *the extensive margin*.
  - Firms' incentive to move across markets with different market sizes independent of firm productivity

  Inconsistent with some evidence for
  - An increase in the production cost leads to less than proportional increase in the price (the pass-through rate < 1)
  - More productive firms have higher markup rates
  - More productive firms have lower pass-through rates
- Melitz-Ottaviano (2008) departs from CES with **Linear Demand System + the outside competitive sector**, which comes with its own restrictions.

This Paper: Melitz under **H.S.A.** Demand System as a framework to study how departing from CES in the direction consistent with the evidence affects the impact of competitive pressures on heterogeneous firms.

Symmetric H.S.A. (Homothetic with a Single Aggregator) DS with Gross Substitutes

Think of a competitive final goods industry generating demand for a continuum of **intermediate inputs** $\omega \in \Omega$, with
CRS production function: $X = X(\mathbf{x})$; $\mathbf{x} = \{x_\omega;\ \omega \in \Omega\} \Leftrightarrow$ **Unit cost function,** $P = P(\mathbf{p})$; $\mathbf{p} = \{p_\omega;\ \omega \in \Omega\}$.

Market share of ω depends *solely* on a single variable, its own price normalized by the *common* price aggregator

$$s_\omega \equiv \frac{p_\omega x_\omega}{\mathbf{px}} = \frac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega} = s\left(\frac{p_\omega}{A(\mathbf{p})}\right), \qquad \text{where} \qquad \int_\Omega s\left(\frac{p_\omega}{A(\mathbf{p})}\right) d\omega \equiv 1.$$

- $s: \mathbb{R}_{++} \to \mathbb{R}_+$: **the market share function**, C^3, decreasing in the **normalized price** $z_\omega \equiv p_\omega / A$ for $s(z_\omega) > 0$ with
  - $\lim_{z \to \bar{z}} s(z) = 0$. If $\bar{z} \equiv \inf\{z > 0 | s(z) = 0\} < \infty$, $\bar{z}A(\mathbf{p})$ is the **choke price.**
- $A = A(\mathbf{p})$: the **common price aggregator** defined implicitly by **the adding-up constraint** $\int_\Omega s(p_\omega / A) d\omega \equiv 1$.
  $A(\mathbf{p})$ linear homogenous in $\mathbf{p}$ for a fixed Ω. A larger Ω reduces $A(\mathbf{p})$.

	CES	$s(z) = \gamma z^{1-\sigma}$;	$\sigma > 1$
Special Cases	**Translog Cost Function**	$s(z) = \gamma \max\{-\ln(z/\bar{z}), 0\}$;	$\bar{z} < \infty$
	Constant Pass Through (CoPaTh)	$s(z) = \gamma \max\left\{\left[\sigma + (1-\sigma) z^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}, 0\right\}$	$0 < \rho < 1$

As $\rho \nearrow 1$, CoPaTh converges to CES with $\bar{z}(\rho) \equiv (\sigma/(\sigma-1))^{\frac{\rho}{1-\rho}} \to \infty$.

$P(\mathbf{p})$ vs. $A(\mathbf{p})$

Definition: $s_\omega \equiv \dfrac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega} = s\left(\dfrac{p_\omega}{A(\mathbf{p})}\right) = s(z_\omega)$ where $\displaystyle\int_\Omega s\left(\frac{p_\omega}{A(\mathbf{p})}\right) d\omega \equiv 1.$

By differentiating the adding-up constraint, $\dfrac{\partial \ln A(\mathbf{p})}{\partial \ln p_\omega} = \dfrac{[\zeta(z_\omega) - 1]s(z_\omega)}{\int_\Omega [\zeta(z_{\omega'}) - 1]s(z_{\omega'})d\omega'} \neq s(z_\omega) = \dfrac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega}$

unless $\zeta(z_\omega)$ is constant, where

Price Elasticity Function: $\zeta(z) \equiv 1 - \dfrac{d \ln s(z)}{d \ln z} \equiv 1 - \mathcal{E}_s(z) > 1 \Leftrightarrow s(z) = \gamma \exp\left[\displaystyle\int_{z_0}^{z} \frac{1-\zeta(\xi)}{\xi} d\xi\right]$; $\displaystyle\lim_{z \to \bar{z}} \zeta(z) = \infty$, if $\bar{z} < \infty$.

By integrating the definition, $\dfrac{A(\mathbf{p})}{P(\mathbf{p})} = c \exp\left[\displaystyle\int_\Omega s\left(\frac{p_\omega}{A(\mathbf{p})}\right) \Phi\left(\frac{p_\omega}{A(\mathbf{p})}\right) d\omega\right]$, where $\Phi(z) \equiv \dfrac{1}{s(z)} \displaystyle\int_z^{\bar{z}} \frac{s(\xi)}{\xi} d\xi$

Note: $A(\mathbf{p})/P(\mathbf{p})$ is not constant, unless CES $\Leftrightarrow \zeta(z) = \sigma \Leftrightarrow s(z) = \gamma z^{1-\sigma} \Leftrightarrow \Phi(z) = 1/(\sigma - 1)$.

- ✓ $A(\mathbf{p})$, the inverse measure of *competitive pressures*, captures *cross price effects* in the DS, the reference price for MC firms
- ✓ $P(\mathbf{p})$, the inverse measure of TFP, captures the *productivity effects* of price changes, the reference price for consumers.
- ✓ $\Phi(z)$, the measure of "love for variety." Matsuyama & Ushchev (2023). $\zeta'(\cdot) \gtreqless 0 \implies \Phi'(\cdot) \lesseqgtr 0$; $\Phi'(\cdot) = 0 \Leftrightarrow \zeta'(\cdot) = 0$.

Note: Our 2017 paper proved the integrability = the quasi-concavity of $P(\mathbf{p})$, iff $\zeta(z) \equiv 1 - \frac{d \ln s(z)}{d \ln z} \equiv 1 - \mathcal{E}_s(z) > 0$.

Why H.S.A.

- **Homothetic** (unlike the linear DS and most other commonly used non-CES DSs)
  - a single measure of market size; the demand composition does not matter.
  - isolate the effect of endogenous markup rate from nonhomotheticity
  - straightforward to use it as a building block in multi-sector models with any upper-tier (incl. nonhomothetic) DS

- **Nonparametric** and **flexible** (unlike **CES** and **translog**, which are special cases)
  - can be used to perform robustness-check for CES
  - allow for (but no need to impose)
    - ✓ the choke price,
    - ✓ **Marshall's 2nd law** (Price elasticity is increasing in price) → more productive firms have higher markup rates
    - ✓ *what we call* **the 3rd law** (the rate of increase in the price elasticity is decreasing in price) → more productive firms have lower pass-through rates.

- **Tractable** due to **Single Aggregator** (unlike **Kimball**, which needs two aggregators), a *sufficient statistic* for competitive pressures, which acts like a *magnifier of firm heterogeneity*
  - guarantee the existence & uniqueness of free-entry equilibrium with firm heterogeneity
  - simple to conduct most comparative statics without *parametric* restrictions on demand or productivity distribution.
  - no need to assume zero overhead cost (unlike MO and ACDR)

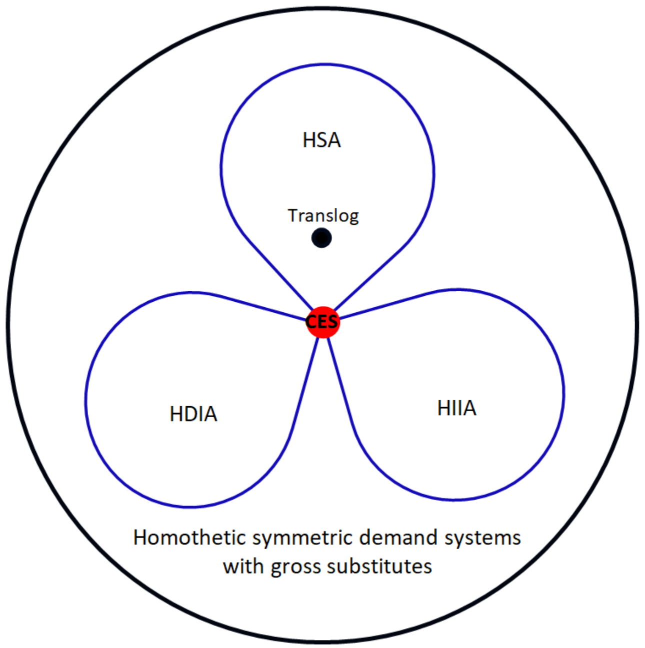
- Defined by **the market share function,** for which data is readily available and easily identifiable.

Three Classes of Homothetic Demand Systems: Matsuyama-Ushchev (2017)

Here we consider a **continuum** of varieties ($\omega \in \Omega$), **gross substitutes**, and **symmetry**

CES	$s_\omega \equiv \frac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega} = f\left(\frac{p_\omega}{P(\mathbf{p})}\right) \Leftrightarrow s_\omega \propto \left(\frac{p_\omega}{P(\mathbf{p})}\right)^{1-\sigma}$	
H.S.A. (Homotheticity with a Single Aggregator)	$s_\omega = s\left(\frac{p_\omega}{A(\mathbf{p})}\right)$,	$\frac{P(\mathbf{p})}{A(\mathbf{p})} \neq c$, unless CES
HDIA (Homotheticity with Direct Implicit Additivity) Kimball is a special case:	$s_\omega = \frac{p_\omega}{P(\mathbf{p})}(\phi')^{-1}\left(\frac{p_\omega}{B(\mathbf{p})}\right)$,	$\frac{P(\mathbf{p})}{B(\mathbf{p})} \neq c$, unless CES
HIIA (Homotheticity with Indirect Implicit Additivity)	$s_\omega = \frac{p_\omega}{C(\mathbf{p})}\theta'\left(\frac{p_\omega}{P(\mathbf{p})}\right)$,	$\frac{P(\mathbf{p})}{C(\mathbf{p})} \neq c$, unless CES

$\phi(\cdot)$ & $\theta(\cdot)$ are both increasing & concave → $(\phi')^{-1}(\cdot)$ & $\theta'(\cdot)$ positive-valued & decreasing.
$A(\cdot), B(\cdot), C(\cdot)$ all determined by the adding-up constraint.



The 3 classes are pairwise disjoint with the sole exception of CES.

Under HDIA(Kimball) and HIIA, unlike HSA

- Two aggregators needed for the market shares. [One aggregator enough for the price elasticity under all 3 classes.]
- The existence and uniqueness of free-entry equilibrium not guaranteed without some strong restrictions on both productivity distribution and the price elasticity function.

Melitz under HSA: Main Results

- **Existence & Uniqueness of Equilibrium:** straightforward under H.S.A.
- **Melitz under CES:** impacts of entry/overhead costs on the masses of entrants/active firms hinges on the sign of the derivative of the elasticity of the pdf of marginal cost; Pareto is the knife-edge! (new results!)
- **Cross-Sectional Implications:** profits and revenues are always higher among more productive.
  - 2nd Law = incomplete pass-through ⇔ the procompetitive effect ⇔ strategic complementarity in pricing.
  - 2nd (3rd) Law → more productive firms have higher markup (lower pass-through) rates.
  - 2nd & 3rd Laws → *hump-shaped* employment; more productive hire less under high overhead.
- **General Equilibrium Comparative Statics**
  - *Entry cost* ↓: 2nd (3rd) Law → markup rates ↓ (pass-through rates ↑) for all firms.
    profits (revenues) decline faster among less productive → a tougher selection.
  - *Overhead cost* ↓: similar effects when the employment is decreasing in firm productivity.
  - *Market size* ↑: 2nd (3rd) Law → markup rates ↓ (pass-through rates ↑) for all firms.
    profits (revenues) ↑ among more productive; ↓ among less productive.
  - *Due to the composition effect*, these changes may *increase* the average markup rate & the aggregate profit share in spite of 2nd Law and *reduce* the average pass-through in spite of 3rd Law; Pareto is the knife-edge *for entry cost* ↑.
- **Sorting of Heterogeneous Firms** across markets that differ in size: Larger markets → more competitive pressures.
  - 2nd Law → more (less) productive go into larger (smaller) markets.
  - *Composition effect*, average markup (pass-through) rates can be *higher* (*lower*) in larger and more competitive markets in spite of 2nd (3rd) Law.

(Highly Selective) Literature Review

Non-CES Demand Systems: Matsuyama (2023) for a survey; **H.S.A. Demand System:** Matsuyama-Ushchev (2017)

MC with Heterogeneous Firms: Melitz (2003) and many others: Melitz-Redding (2015) for a survey

MC under non-CES demand systems: Thisse-Ushchev (2018) for a survey

- *Nonhomothetic non-CES:*
  - $U = \int_{\Omega} u(x_\omega) d\omega$: Dixit-Stiglitz (77), Behrens-Murata (07), ZKPT (12), Mrázová-Neary(17), Dhingra-Morrow (19); ACDR (19)
  - *Linear-demand system with the outside sector:* Ottaviano-Tabuchi-Thisse (2002), Melitz-Ottaviano (2008)
- *Homothetic non-CES:* Feenstra (2003), Kimball (1995), Matsuyama-Ushchev (2020a,b, 2023)
- *H.S.A.* Matsuyama-Ushchev (2022), Kasahara-Sugita (2020), Grossman-Helpman-Lhuiller (2021), Fujiwara-Matsuyama (2022), Baqaee-Fahri-Sangani (2023)

Empirical Evidence: *The 2nd Law:* DeLoecker-Goldberg (14), Burstein-Gopinath (14); *The 3rd Law:* Berman et.al.(12), Amiti et.al.(19), *Market Size Effects:* Campbell-Hopenhayn(05); *Rise of markup:* Autor et.al.(20), DeLoecker et.al.(20)

Selection of Heterogeneous Firms through Competitive Pressures
Melitz-Ottaviano (2008), Baqaee-Fahri-Sangani (2023), Edmond-Midrigan-Xu (2023)

Sorting of Heterogeneous Firms Across Markets:

- *Reduced Form/Partial Equilibrium;* Mrázová-Neary (2019), Nocke (2006)
- *General Equilibrium:* Baldwin-Okubo (2006), Behrens-Duranton-RobertNicoud (2014), Davis-Dingel (2019), Gaubert (2018), Kokovin et.al. (2022)

Log-Super(Sub)modularity: Costinot (2009), Costinot-Vogel (2015)

Selection of Heterogeneous Firms: A Single-Market Setting

A Static, Closed Economy Version of Melitz (2003), extended to H.S.A.

Households: supply labor (numeraire) by L, consume **the final good** by X with the budget constraint, $PX = L$.

Final Good Producers: competitive, assemble **intermediate inputs** $\omega \in \Omega$, using **CRS technology**

Production Function: $$X = X(\mathbf{x}) \equiv \min_{\mathbf{p}} \left\{ \mathbf{px} = \int_\Omega p_\omega x_\omega d\omega \,\middle|\, P(\mathbf{p}) \geq 1 \right\}$$

Unit Cost Function: $$P = P(\mathbf{p}) \equiv \min_{\mathbf{x}} \left\{ \mathbf{px} = \int_\Omega p_\omega x_\omega d\omega \,\middle|\, X(\mathbf{x}) \geq 1 \right\}$$

Note: Both $X(\mathbf{x})$ and $P(\mathbf{p})$ can be a primitive of CRS technology, as long as linear homogeneity, monotonicity and quasi-concavity are satisfied.

Demand Curve for ω: $x_\omega = X(\mathbf{x}) \frac{\partial P(\mathbf{p})}{\partial p_\omega}$; **Inverse Demand Curve for ω:** $p_\omega = P(\mathbf{p}) \frac{\partial X(\mathbf{x})}{\partial x_\omega}$

Market Size: $\mathbf{px} = P(\mathbf{p})X(\mathbf{x}) = L$

Note: This is due to the one-market setting. In a multi-market extension later, size of each market differs from L.

Symmetric H.S.A. (Homothetic with a Single Aggregator) with Gross Substitutes

Market Share of ω depends *solely* on a single variable, its own price normalized by the *common* price aggregator

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Monopolistically Competitive Intermediate Inputs Producers $\omega \in \Omega$

Timing: the same with Melitz.

- Sunk cost of entry, $F_e > 0$. (All costs are paid in labor.)
- Each entrant draws its (quality-adjusted) marginal cost $\psi \sim G(\cdot) \in C^3$ with $G'(\psi) = g(\psi) > 0$ on $(\underline{\psi}, \overline{\psi}) \subseteq (0, \infty)$.
  $\mathcal{E}_G(\psi) \equiv \psi g(\psi)/G(\psi) \in C^2$ and $\mathcal{E}_g(\psi) \equiv \psi g'(\psi)/g(\psi) \in C^1$.
  MC firms are ex-post heterogeneous *only* in ψ, or equivalently, in (quality-adjusted) productivity, $1/\psi = \varphi \sim 1 - G(1/\varphi)$ with density $g(1/\varphi)/\varphi^2 > 0$ on $(\underline{\varphi}, \overline{\varphi}) \subseteq (0, \infty)$.
- Each firm decides either to exit without producing or to stay & produce with an overhead cost, $F > 0$.
- Firms that stay will sell their products at the profit-maximizing prices.

Pricing Behaviors of MC firms after drawing ψ_ω**:** Each firm takes $A = A(\mathbf{p})$ and $\mathbf{px} = L$ given.

$$\max_{p_\omega}(p_\omega - \psi_\omega)x_\omega = \max_{\psi_\omega < p_\omega < \bar{z}A}\left(1 - \frac{\psi_\omega}{p_\omega}\right) s\left(\frac{p_\omega}{A}\right) L = \max_{\psi_\omega/A < z_\omega < \bar{z}}\left(1 - \frac{\psi_\omega/A}{z_\omega}\right) s(z_\omega) L$$

where $z_\omega \equiv p_\omega/A$ is the normalized price.

FOC:

$$z_\omega\left[1-\frac{1}{\zeta(z_\omega)}\right]=\frac{\psi_\omega}{A}$$

Price Elasticity Function

$$\zeta(z)\equiv 1-\frac{d\ln s(z)}{d\ln z}\equiv 1-\mathcal{E}_s(z)>1,$$

for $z\in(0,\bar{z})$ with $\lim_{z\to\bar{z}}\zeta(z)=-\lim_{z\to\bar{z}}\mathcal{E}_s(z)=\infty$, if $\bar{z}$ is finite. The markup rate is $\zeta(z_\omega)/(\zeta(z_\omega)-1)$.

We maintain the following ***regularity*** assumption *for ease of exposition*.

(A1): For all $z\in(0,\bar{z})$,

$$\mathcal{E}_{z(\zeta-1)/\zeta}(z)>0\Leftrightarrow\mathcal{E}_{\zeta/(\zeta-1)}(z)<1\Leftrightarrow\mathcal{E}_{s/\zeta}(z)=\mathcal{E}_s(z)-\mathcal{E}_\zeta(z)<0$$

- **(A1)** means that $\zeta(z)/(\zeta(z)-1)$ cannot go up as fast as z.
  →**(A1)** holds with decreasing $\zeta(\cdot)/(\zeta(\cdot)-1)$ ↔ increasing $\zeta(\cdot)$, i.e., under A2 (**Marshall's 2nd Law**.
- **(A1)** means the marginal revenue is strictly increasing in p_ω (hence strictly decreasing in x_ω)
  → FOC determines the profit maximizing z_ω as an increasing C^2 function of ψ_ω/A.
  → Firms with the same ψ set the same price, earn the same profit → we index firms by ψ, as p_ψ, $z_\psi\equiv p_\psi/A$.
- **(A1)** ensures that the maximized profit $s(\cdot)L/\zeta(\cdot)$ is a decreasing C^2 function of ψ_ω/A.

Without (A1), the maximizing price would be piecewise-continuous (i.e., the price would jump up at some values of ψ) and the maximized profit would be piecewise-differentiable, which would complicate the analysis.

Monopolistic Competition under H.S.A.: Markup and Pass-Through Rates

Lerner Pricing Formula: $$z_\psi\left[1-\frac{1}{\zeta(z_\psi)}\right]=\frac{\psi}{A}$$

Under **A1**, LHS is strictly increasing, so the Inverse Function Theorem allows us to rewrite it as

Normalized Price: $$\frac{p_\psi}{A}\equiv z_\psi=Z\left(\frac{\psi}{A}\right)\in(\psi/A\,,\bar{z}),Z'(\cdot)>0;$$

Price Elasticity: $$\zeta(z_\psi)=\zeta\left(Z\left(\frac{\psi}{A}\right)\right)\equiv\sigma\left(\frac{\psi}{A}\right)>1;$$ **Markup Rate:** $$\mu_\psi\equiv\frac{p_\psi}{\psi}=\frac{\sigma(\psi/A)}{\sigma(\psi/A)-1}\equiv\mu\left(\frac{\psi}{A}\right)>1$$

satisfying

$$\frac{1}{\sigma(\psi/A)}+\frac{1}{\mu(\psi/A)}=1\Leftrightarrow\left[\sigma\left(\frac{\psi}{A}\right)-1\right]\left[\mu\left(\frac{\psi}{A}\right)-1\right]=1$$

Pass-Through Rate: $$\rho_\psi\equiv\frac{\partial\ln p_\psi}{\partial\ln\psi}=\mathcal{E}_Z\left(\frac{\psi}{A}\right)\equiv\rho\left(\frac{\psi}{A}\right)=1+\mathcal{E}_\mu\left(\frac{\psi}{A}\right)=1-\frac{\mathcal{E}_\sigma(\psi/A)}{\sigma(\psi/A)-1}>0$$

- Normalized price, and markup rate, all C^2 functions of the *normalized cost,* ψ/A only.
  - $Z'(\cdot)>0$; always strictly increasing in ψ/A; Markup rate, strictly decreasing in ψ/A under A2
- Pass-through rate, a C^1 function of ψ/A only, strictly increasing in ψ/A under strong A3**.**
- Market size affects the pricing behaviors of firms only through its effects on A.
- More competitive pressures, a lower A, act like a magnifier of firm heterogeneity.

Under CES, $\sigma(\cdot)=\sigma;\ \mu(\cdot)=\sigma/(\sigma-1)=\mu;\ \rho(\cdot)=1.$

Revenue, Profit, and Employment

Revenue $$R_\psi = s(z_\psi)L = s\left(Z\left(\frac{\psi}{A}\right)\right)L \equiv r\left(\frac{\psi}{A}\right)L \quad \Longrightarrow \quad \mathcal{E}_r\left(\frac{\psi}{A}\right) = \left[1 - \sigma\left(\frac{\psi}{A}\right)\right]\rho\left(\frac{\psi}{A}\right) < 0$$

(Gross) Profit $$\Pi_\psi = \frac{r(\psi/A)}{\sigma(\psi/A)}L \equiv \pi\left(\frac{\psi}{A}\right)L \quad \Longrightarrow \quad \mathcal{E}_\pi\left(\frac{\psi}{A}\right) = 1 - \sigma\left(\frac{\psi}{A}\right) < 0$$

(Variable) Employment $$\psi x_\psi = \frac{r(\psi/A)}{\mu(\psi/A)}L \equiv \ell\left(\frac{\psi}{A}\right)L \quad \Longrightarrow \quad \mathcal{E}_\ell\left(\frac{\psi}{A}\right) = 1 - \sigma\left(\frac{\psi}{A}\right)\rho\left(\frac{\psi}{A}\right) \lesseqgtr 0$$

- Revenue $r(\psi/A)L$, profit $\pi(\psi/A)L$, employment $\ell(\psi/A)L$ all C^2 functions of ψ/A, multiplied by **market size** L.
- Their elasticities $\mathcal{E}_r(\cdot), \mathcal{E}_\pi(\cdot)$ and $\mathcal{E}_\ell(\cdot)$ depend solely on $\sigma(\cdot)$ and $\rho(\cdot)$, hence all C^1 functions of ψ/A only.

  More competitive pressures, a lower A, act like a magnifier of firm heterogeneity.

  Market size affects the relative profit, revenue, and employment across firms only through its effects on A.

Under CES, $r(\cdot)/\pi(\cdot) = \sigma$; $r(\cdot)/\ell(\cdot) = \mu = \sigma/(\sigma - 1) \Longrightarrow \mathcal{E}_r(\cdot) = \mathcal{E}_\pi(\cdot) = \mathcal{E}_\ell(\cdot) = 1 - \sigma < 0$.

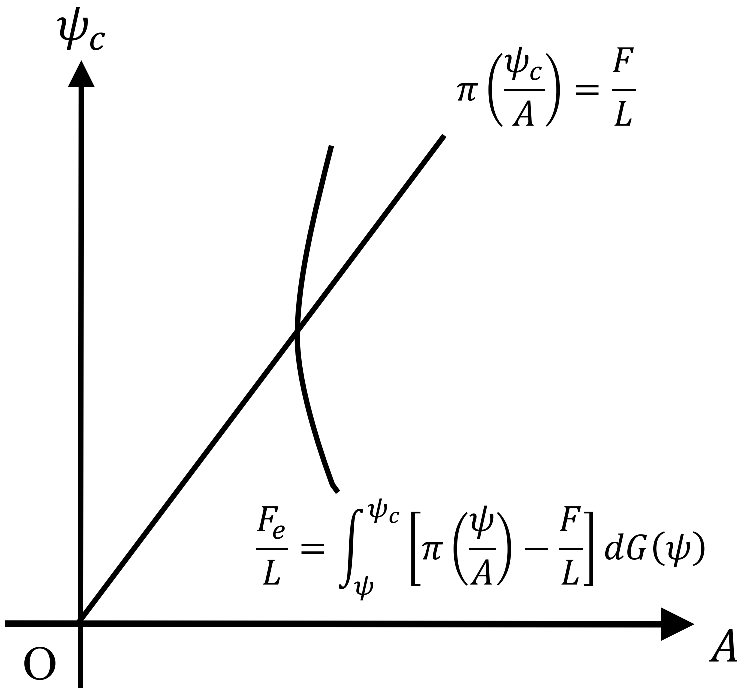
- Both revenue $r(\psi/A)L$ and profit $\pi(\psi/A)L$ are always strictly decreasing in ψ/A.
- Employment $\ell(\psi/A)L$ may be nonmonotonic in ψ/A.
  - If the markup rate declines with ψ/A, employment cannot decline as fast as the revenue.
  - If the markup rate declines faster than the revenue, the employment is *increasing* in ψ/A.

General Equilibrium: Existence and Uniqueness: Assume $F + F_e < \pi(0)L$.

Cutoff Rule: Stay if $\psi < \psi_c$; exit if $\psi > \psi_c$, where

$$\max_{\psi_c} \int_{\underline{\psi}}^{\psi_c} \left[\pi\left(\frac{\psi}{A}\right)L - F\right] dG(\psi) \Longrightarrow \pi\left(\frac{\psi_c}{A}\right)L = F$$

positively-sloped. $A \downarrow$ (more competitive pressures) $\Longrightarrow \psi_c \downarrow$ (tougher selection)
rotate clockwise, as $F/L \uparrow$ (higher overhead/market size) $\Longrightarrow \psi_c/A \downarrow$.



Free Entry Condition:

$$F_e = \int_{\underline{\psi}}^{\psi_c} \left[\pi\left(\frac{\psi}{A}\right)L - F\right] dG(\psi)$$

shift to the left as $F_e \downarrow$ (lower entry cost) $\Longrightarrow A \downarrow$ (more competitive pressures).

$A = A(\mathbf{p})$ and ψ_c: uniquely determined as C^2 functions of F_e/L & F/L with the interior solution, $0 < G(\psi_c) < 1$ for

$$0 < \frac{F_e}{L} < \int_{\underline{\psi}}^{\overline{\psi}} \left[\pi\left(\pi^{-1}\left(\frac{F}{L}\right)\frac{\psi}{\overline{\psi}}\right) - \frac{F}{L}\right] dG(\psi),$$

which holds for a sufficiently small $F_e > 0$ with no further restrictions on $G(\cdot)$ and $s(\cdot)$.
(This unique existence proof does not assume A2 and hence applies also to the Melitz model under CES.)

Equilibrium Mass of Firms. From the adding-up constraint, $1 \equiv \int_\Omega s(p_\omega/A)d\omega = M\int_{\underline{\psi}}^{\psi_c} r(\psi/A)dG(\psi)$,

Mass of entrants

$$M = \left[\int_{\underline{\psi}}^{\psi_c} r\left(\frac{\psi}{A}\right) dG(\psi)\right]^{-1} = \left[\int_{\underline{\xi}}^{1} r\left(\pi^{-1}\left(\frac{F}{L}\right)\xi\right) dG(\psi_c\xi)\right]^{-1} > 0$$

Mass of active firms
= the measure of Ω

$$MG(\psi_c) = \left[\int_{\underline{\psi}}^{\psi_c} r\left(\frac{\psi}{A}\right) \frac{dG(\psi)}{G(\psi_c)}\right]^{-1} = \left[\int_{\underline{\xi}}^{1} r\left(\pi^{-1}\left(\frac{F}{L}\right)\xi\right) d\tilde{G}(\xi;\psi_c)\right]^{-1} > 0$$

where $\tilde{G}(\xi;\psi_c) \equiv \frac{G(\psi_c\xi)}{G(\psi_c)}$ is the cdf of $\xi \equiv \psi/\psi_c$, conditional on $\underline{\xi} \equiv \underline{\psi}/\psi_c < \xi \leq 1$.

Lemma 1: $\mathcal{E}_g'(\psi) < 0 \Longrightarrow \mathcal{E}_G'(\psi) < 0$; $\mathcal{E}_g'(\psi) \geq 0 \Longrightarrow \mathcal{E}_G'(\psi) \geq 0$ with some boundary conditions.
Lemma 2: A lower ψ_c shifts $\tilde{G}(\xi;\psi_c)$ to the right (left) in MLR if $\mathcal{E}_g'(\psi) < (>)0$ and in FSD if $\mathcal{E}_G'(\psi) < (>)0$.

- Some evidence for $\mathcal{E}_g'(\psi) > 0 \Longrightarrow \psi_c \downarrow$ (tougher selection) shifts $\tilde{G}(\xi;\psi_c)$ to the left.
- Pareto-productivity, $G(\psi) = (\psi/\bar{\psi})^\kappa \Longrightarrow \mathcal{E}_g'(\psi) = \mathcal{E}_G'(\psi) = 0 \Longrightarrow \tilde{G}(\xi;\psi_c)$ is independent of ψ_c.
- Fréchet, Weibull, Lognormal; $\mathcal{E}_g'(\psi) < 0 \Longrightarrow \mathcal{E}_G'(\psi) < 0 \Longrightarrow \psi_c \downarrow$ (tougher selection) shifts $\tilde{G}(\xi;\psi_c)$ to the right.

Lemma 4: The integrals in the equilibrium conditions are finite and hence the equilibrium is well-defined, if $\underline{\psi} > 0 \Leftrightarrow \overline{\varphi} < \infty$ or $1 + \lim_{z\to 0}\zeta(z) < 2 + \lim_{\psi\to 0}\mathcal{E}_g(\psi) = -\lim_{\varphi\to\infty}\mathcal{E}_f(\varphi) < \infty$ for $\underline{\psi} = 0 \Leftrightarrow \overline{\varphi} = \infty$.

Equilibrium can be solved recursively under H.S.A.!!
Under HDIA/HIIA, one needs to solve the 3 equations simultaneously for 3 variables, ψ_c & the two price aggregates.

Aggregate Labor Cost and Profit Shares and TFP

Notations:

The $w(\cdot)$-weighted average of $f(\cdot)$ among the active firms, $\psi \in (\underline{\psi}, \psi_c)$	$\mathbb{E}_w(f) \equiv \dfrac{\int_{\underline{\psi}}^{\psi_c} f\left(\frac{\psi}{A}\right) w\left(\frac{\psi}{A}\right) dG(\psi)}{\int_{\underline{\psi}}^{\psi_c} w\left(\frac{\psi}{A}\right) dG(\psi)}.$
The unweighted average of $f(\cdot)$ among the active firms, $\psi \in (\underline{\psi}, \psi_c)$	$\mathbb{E}_1(f) \equiv \dfrac{\int_{\underline{\psi}}^{\psi_c} f\left(\frac{\psi}{A}\right) dG(\psi)}{\int_{\underline{\psi}}^{\psi_c} dG(\psi)}.$

$$\Rightarrow \mathbb{E}_w\left(\frac{f}{w}\right) = \frac{\mathbb{E}_1(f)}{\mathbb{E}_1(w)} = \left[\mathbb{E}_f\left(\frac{w}{f}\right)\right]^{-1}.$$

Then,

Aggregate TFP	$\ln\left(\frac{X}{L}\right) = \ln\left(\frac{1}{P}\right) = \ln\left(\frac{c}{A}\right) + \mathbb{E}_r[\Phi \circ Z]$
Aggregate Labor Cost Share (Average inverse markup rate)	$\frac{\mathbb{E}_1(\ell)}{\mathbb{E}_1(r)} = \mathbb{E}_r\left(\frac{1}{\mu}\right) = 1 - \left[\mathbb{E}_\pi\left(\frac{\mu}{\mu - 1}\right)\right]^{-1} = \frac{1}{\mathbb{E}_\ell(\mu)}$
Aggregate Profit Share (Average inverse price elasticity)	$\frac{\mathbb{E}_1(\pi)}{\mathbb{E}_1(r)} = \mathbb{E}_r\left(\frac{1}{\sigma}\right) = \frac{1}{\mathbb{E}_\pi(\sigma)} = 1 - \left[\mathbb{E}_\ell\left(\frac{\sigma}{\sigma - 1}\right)\right]^{-1}$

by applying the above formulae to $\pi(\cdot)/r(\cdot) = 1 - \ell(\cdot)/r(\cdot) = 1/\sigma(\cdot) = 1 - 1/\mu(\cdot)$,

CES Benchmark: Revisiting Melitz

CES Benchmark: For all $z \in (0,\infty)$, $\zeta(z) = \sigma > 1 \Leftrightarrow s(z) = \gamma z^{1-\sigma}$.

Pricing: $$p_\psi\left(1-\frac{1}{\sigma}\right) = \psi \Leftrightarrow \mu\left(\frac{\psi}{A}\right) = \frac{\sigma}{\sigma-1} > 1 \Rightarrow \rho\left(\frac{\psi}{A}\right) = 1$$

Markup rate constant; Pass-through rate equal to one.

Cutoff Rule: $$c_0 L\left(\frac{\psi_c}{A}\right)^{1-\sigma} = F,$$

Free Entry Condition: $$\int_{\underline{\psi}}^{\psi_c}\left[c_0 L\left(\frac{\psi}{A}\right)^{1-\sigma} - F\right]dG(\psi) = F_e,$$

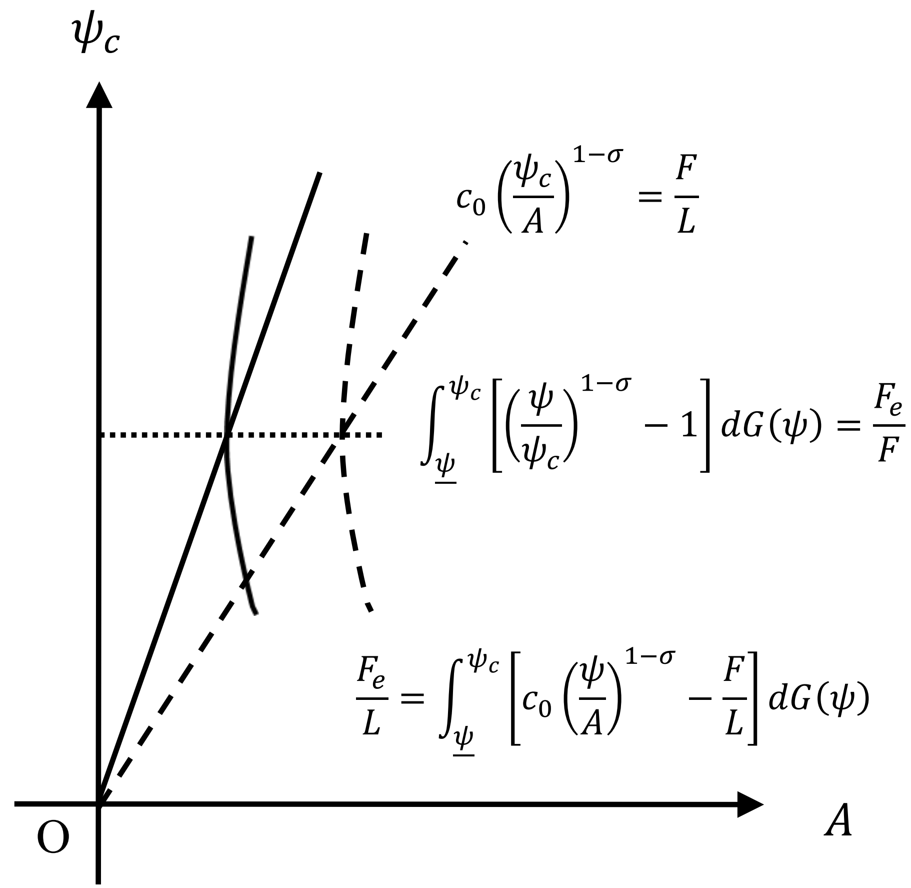
with $c_0 > 0$. As L changes, the intersection moves along

$$\int_{\underline{\psi}}^{\psi_c}\left[\left(\frac{\psi}{\psi_c}\right)^{1-\sigma} - 1\right]dG(\psi) = \frac{F_e}{F}$$

$F_e/F \downarrow$ and a FSD shift of $G(\cdot)$ *to the left* $\Longrightarrow \psi_c \downarrow$ (tougher selection).

ψ_c unaffected by L, and independent of A.

$$A = \psi_c\left(\frac{c_0 L}{F}\right)^{\frac{1}{1-\sigma}} = \left(\frac{c_0 L}{F_e}\int_{\underline{\psi}}^{\psi_c}[(\psi)^{1-\sigma} - (\psi_c)^{1-\sigma}]dG(\psi)\right)^{\frac{1}{1-\sigma}}.$$



$L \uparrow$, $F_e \downarrow$, $F \downarrow$, a FSD shift of $G(\cdot)$ to the left $\Longrightarrow A \downarrow$ (more competitive pressures)

CES Benchmark (Continue)

Revenue:

$$r\left(\frac{\psi}{A}\right)L = \sigma c_0 L\left(\frac{\psi}{A}\right)^{1-\sigma} = \sigma F\left(\frac{\psi}{\psi_c}\right)^{1-\sigma} \geq \sigma F$$

(Gross) Profi:

$$\pi\left(\frac{\psi}{A}\right)L = c_0 L\left(\frac{\psi}{A}\right)^{1-\sigma} = F\left(\frac{\psi}{\psi_c}\right)^{1-\sigma} \geq F$$

(Variable) Employment:

$$\ell\left(\frac{\psi}{A}\right)L = (\sigma-1)c_0 L\left(\frac{\psi}{A}\right)^{1-\sigma} = (\sigma-1)F\left(\frac{\psi}{\psi_c}\right)^{1-\sigma} \geq (\sigma-1)F$$

All decreasing **power** functions of ψ with

$$\mathcal{E}_r\left(\frac{\psi}{A}\right) = \mathcal{E}_\pi\left(\frac{\psi}{A}\right) = \mathcal{E}_\ell\left(\frac{\psi}{A}\right) = 1-\sigma < 0.$$

Relative size of two firms with $\psi,\ \psi' \in (\underline{\psi}, \psi_c)$, whether measured in the profit, employment, and revenue, unaffected by $L, F_e, F, G(\cdot)$, as well as A and ψ_c, and thus never change across equilibriums.

CES Benchmark (Continue)

Mass of entrants

$$M = \frac{L/\sigma}{F_e + G(\psi_c)F} = \frac{L}{\sigma F_e}\left[1 - \frac{1}{H(\psi_c)}\right]$$

Mass of active firms

$$MG(\psi_c) = \frac{L/\sigma}{F_e/G(\psi_c) + F} = \frac{L}{H(\psi_c)\sigma F}$$

where $H(\psi_c) \equiv \int_{\underline{\xi}}^{1} (\xi)^{1-\sigma} \tilde{G}(\xi;\psi_c)$. Since $(\xi)^{1-\sigma}$ is decreasing, $H'(\psi_c) > (<)0$ if $\mathcal{E}'_G(\psi) < (>)0$ (Lemma 2).

Hence,

Proposition 1: Under CES,
- $L \uparrow$ keeps ψ_c unaffected; increases both M and $MG(\psi_c)$ *proportionately;*
- $F_e \downarrow$ reduces ψ_c; increases M; increases (decreases) $MG(\psi_c)$ if $\mathcal{E}'_G(\psi) < (>)0$;
- $F \downarrow$ increases ψ_c; increases $MG(\psi_c)$; increases (decreases) M if $\mathcal{E}'_G(\psi) < (>)0$;

A FSD shift of $G(\cdot)$ to the left reduces ψ_c with ambiguous effects on M and $MG(\psi_c)$, even if $G(\cdot)$ is a power.

Effects of Market Size *L* under CES:
- No effect on the markup rate.
- No effect on the cutoff, ψ_c
- No effect on the distribution of productivity, revenue, and employment across firms.
- Masses of entrants and of active firms change ***proportionately.*** All adjustments at ***the extensive margin.***

Cross-Sectional Implications under 2^{nd} and 3^{rd} Laws

Marshall's 2nd Law (A2)

(A2): $\zeta'(z) > 0$ for all $z \in (0, \bar{z}) \Leftrightarrow \sigma'(\psi/A) > 0$ for all $\psi/A \in (0, \bar{z})$

Note: **A2 ⇒ A1.**

Lemma 5: For a positive-valued function of a single variable, $\psi/A > 0$,

$$sgn\left\{\frac{\partial^2 \ln f(\psi/A)}{\partial\psi\partial A}\right\} = -sgn\left\{\mathcal{E}_f'\left(\frac{\psi}{A}\right)\right\} = -sgn\left\{\frac{d^2 \ln f\left(e^{\ln(\psi/A)}\right)}{(d\ln(\psi/A))^2}\right\}$$

$f(\psi/A)$ *log-super(sub)modular* in ψ & $A \Leftrightarrow \mathcal{E}_f'(\cdot) < (>) 0 \Leftrightarrow \ln f\left(e^{\ln(\psi/A)}\right)$ concave (convex) in $\ln(\psi/A)$

Proposition 2: Under **A2,**

Incomplete Pass-Through $$0 < \rho\left(\frac{\psi}{A}\right) = 1 + \mathcal{E}_\mu\left(\frac{\psi}{A}\right) = 1 - \mathcal{E}_{1/\mu}\left(\frac{\psi}{A}\right) < 1$$

Less efficient firms operate at more elastic parts of demand and have lower markup rates

Procompetitive Effect/ Strategic Complementarity in Pricing $$\frac{\partial \ln p_\psi}{\partial \ln A} = 1 - \rho\left(\frac{\psi}{A}\right) = -\mathcal{E}_\mu\left(\frac{\psi}{A}\right) = \mathcal{E}_{1/\mu}\left(\frac{\psi}{A}\right) > 0$$

More competitive pressures ($A \downarrow$ due to entry or lower prices of competing products) → lower prices/markup rates.

Strict Log-Supermodular Profit: $$\mathcal{E}_\pi'\left(\frac{\psi}{A}\right) < 0 \Leftrightarrow \frac{\partial^2 \ln \pi(\psi/A)}{\partial\psi\partial A} > 0$$

More competitive pressures ($A \downarrow$) → a proportionately larger decline in the profit among high-ψ firms → a larger dispersion of the profit across firms; more concentration of profits among the productive.

Marshall's 3rd Law (A3):

(A3) (A3): Weak (Strong) Marshall's 3rd Law of demand. For all $z \in (0, \bar{z})$,

$$\mathcal{E}'_{\zeta/(\zeta-1)}(z) = -\frac{d}{dz}\left(\frac{z\zeta'(z)}{[\zeta(z)-1]\zeta(z)}\right) \geq (>)0 \iff \rho'\left(\frac{\psi}{A}\right) = \mathcal{E}'_Z\left(\frac{\psi}{A}\right) = \mathcal{E}'_\mu\left(\frac{\psi}{A}\right) \geq (>)0$$

Strong A3 → The markup rate declines at the lower rate for higher z → The pass-through rate higher for higher ψ.

- **A3** has some empirical support. Translog violates **A3**. CoPaTh satisfies Weak **A3** but not **Strong A3.**

Proposition 3: Under **A3(A3),**

Weak (Strict) Log-Submodular Markup Rate: $\mathcal{E}'_Z\left(\frac{\psi}{A}\right) = \rho'\left(\frac{\psi}{A}\right) = \mathcal{E}'_\mu\left(\frac{\psi}{A}\right) \geq (>) < 0 \iff \frac{\partial^2 \ln\left(Z(\psi/A)\right)}{\partial\psi\partial A} = \frac{\partial^2 \ln \mu(\psi/A)}{\partial\psi\partial A} \leq (<)0,$

For the strict case, more competitive pressures ($A \downarrow$) → proportionately smaller rate decline among high-ψ firms.
→ a smaller dispersion of the markup rate across firms.

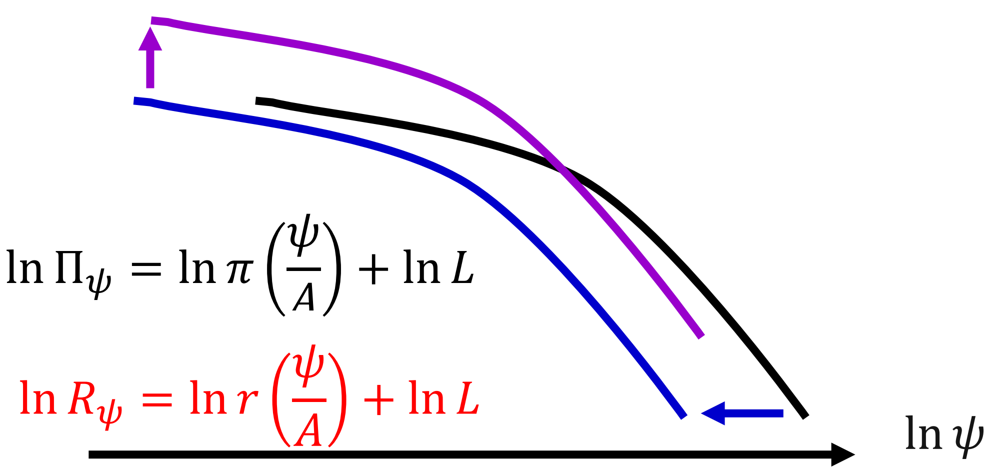
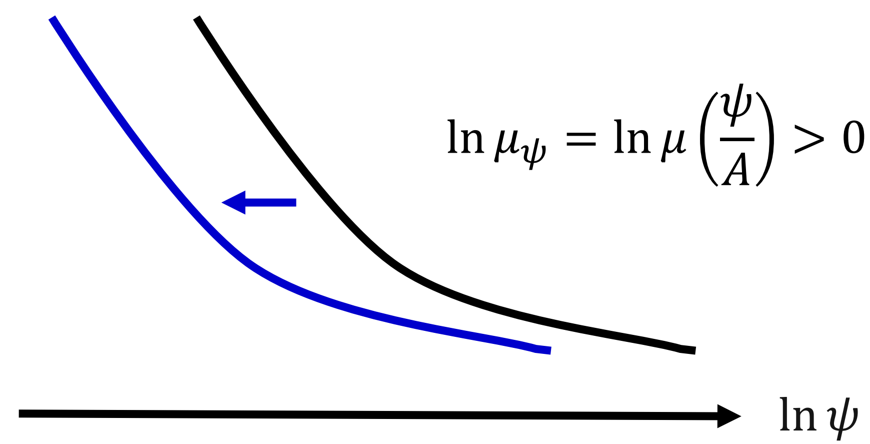
Under **A2+A3**

Strict Log-Supermodular Revenue: $\mathcal{E}'_r\left(\frac{\psi}{A}\right) < 0 \iff \frac{\partial^2 \ln r(\psi/A)}{\partial\psi\partial A} > 0$

Strict Log-Supermodular Employment: $\mathcal{E}'_\ell\left(\frac{\psi}{A}\right) = \mathcal{E}'_r\left(\frac{\psi}{A}\right) - \mathcal{E}'_\mu\left(\frac{\psi}{A}\right) < 0 \iff \frac{\partial^2 \ln \ell(\psi/A)}{\partial\psi\partial A} > 0.$

More competitive pressures ($A \downarrow$) → proportionately larger decline in the revenue among high-ψ firms
→ a larger dispersion of the revenue across firms; more concentration of revenue among the productive.

A2+A3: Cross-Sectional Implications of $A\downarrow$ on Profit and Markup Rate

Profit (Revenue) Function: $\Pi_\psi = \pi(\psi/A)L$; $R_\psi = r(\psi/A)L$	**Markup Rate Function:** $\mu_\psi = \mu(\psi/A) > 1$
• *always* decreasing in ψ • strictly log-supermodular *under A2 (Weak A3)* → $A\downarrow$ with L fixed shifts down with a steeper slope at each ψ; → $A\downarrow$ due to $L\uparrow$, a parallel shift up, a *single-crossing*	• decreasing in ψ *under A2* • weakly log-submodular *under Weak A3* • strictly log-submodular *under Strong A3* → $A\downarrow$ shifts down with a flatter slope at each ψ
	

✓ With $\ln\psi$ in the horizontal axis, $A\downarrow$ causes a parallel leftward shift of the graphs in these figures.

✓ $f(\psi/A)$ is strictly log-super(sub)modular in ψ and A iff $\ln f(e^x)$ is concave(convex) in x.

A2+A3: More Cross-Sectional Implications

Lemma 6: Under A2 and the weak A3, $\lim_{\psi/A\to 0}\rho(\psi/A)\sigma(\psi/A) < 1 < \lim_{\psi/A\to\bar{z}}\rho(\psi/A)\sigma(\psi/A)$.

Since **A2+A3** also implies $\mathcal{E}'_\ell(\psi/A) < 0$,

Proposition 4: Under **A2** and the wea**k A3,** the employment function, $\ell(\psi/A) = \frac{r(\psi/A)}{\mu(\psi/A)}$ is hump-shaped, with its unique peak is reached at, $\hat{z} \equiv Z\left(\hat{\psi}/A\right) < \bar{z}$, where

$$\mathcal{E}_{s(\zeta-1)/\zeta}(\hat{z}) = 0 \Leftrightarrow \frac{\hat{z}\zeta'(\hat{z})}{\zeta(\hat{z})} = [\zeta(\hat{z}) - 1]^2 \Leftrightarrow \mathcal{E}_\ell\left(\frac{\hat{\psi}}{A}\right) = 0 \Leftrightarrow \rho\left(\frac{\hat{\psi}}{A}\right)\sigma\left(\frac{\hat{\psi}}{A}\right) = 1.$$

A2+A3 are sufficient but not necessary for being hump-shaped.

Corollary of Proposition 4: Employments across active firms are

- increasing in ψ if $\psi_c < \hat{\psi} \Leftrightarrow F/L > \pi\left(\hat{\psi}/A\right) = \pi\left(Z^{-1}(\hat{z})\right)$;

This occurs when the overhead/market size ratio is sufficiently high.

- hump-shaped in ψ if $\underline{\psi} < \hat{\psi} < \psi_c \Leftrightarrow F/L < \pi\left(\hat{\psi}/A\right) = \pi\left(Z^{-1}(\hat{z})\right)$ & $A > \underline{\psi}/Z^{-1}(\hat{z})$.

Employments are decreasing among the most productive firms.

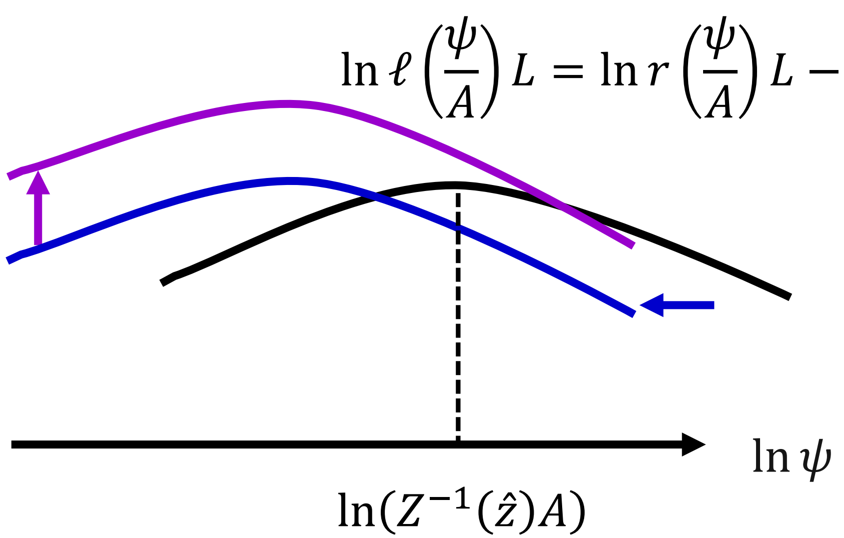
- decreasing in ψ, if $\hat{\psi} < \underline{\psi} \Leftrightarrow A < \underline{\psi}/Z^{-1}(\hat{z})$, which is possible only if $\underline{\psi} > 0$.

Proposition 5: Suppose that A2 and the strong A3 hold, so that $0 < \rho(\psi/A) < 1$ and $\rho(\psi/A)$ is strictly increasing. Then, $\rho(\psi/A)$ is strictly log-submodular for all $\psi/A < \bar{z}$ with a sufficiently small $\bar{z}$.

Employment Function: $\ell(\psi/A)L = r(\psi/A)L/\mu(\psi/A)$

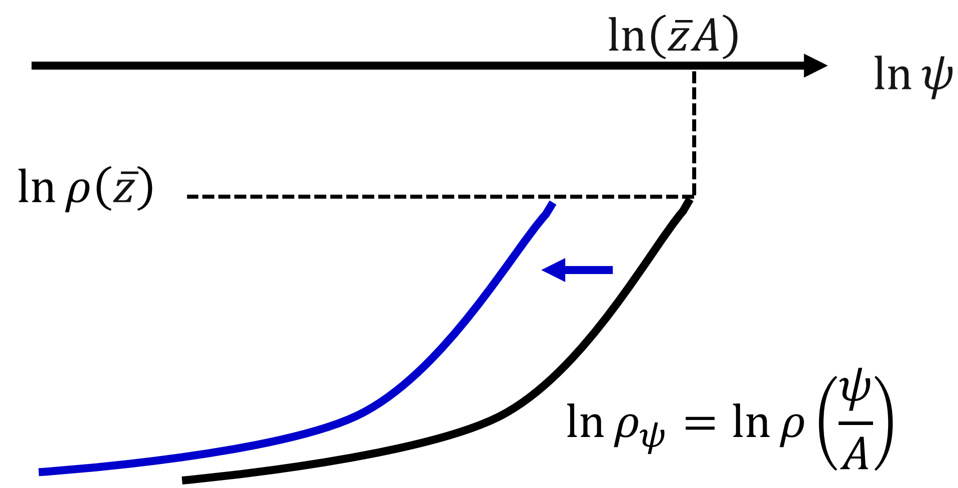
- *Hump-shaped* in ψ *under A2 and weak A3.*
  → $A\downarrow$ shifts up (down) for a low (high) ψ with $A\downarrow$
- Strictly log-supermodular *under Weak A3*
  for $A\downarrow$ with a fixed L; for $A\downarrow$ caused by $L\uparrow$

Single-crossing even with a fixed L



Pass-Through Rate Function: $\rho_\psi = \rho(\psi/A)$

- $\rho(\psi/A) < 1$ *under A2, hence it cannot be* strictly log-submodular for a higher range of ψ/A
- Strictly increasing in ψ *under Strong A3*
- Strictly log-submodular for a lower range of ψ/A *under A2 and Strong A3* ⇒ $A\downarrow$ shifts up with a steeper slope at each ψ *with a small enough* $\bar{z}$.



In summary, more competitive pressures ($A\downarrow$)

- $\mu(\psi/A)\downarrow$ under A2 & $\rho(\psi/A)\uparrow$ under strong A3
- Profit, Revenue, Employment become more concentrated among the most productive.

Comparative Statics: General Equilibrium Effects

Comparative Statics: General Equilibrium Effects of F_e, L , and F on ψ_c and A

Proposition 6:

$$\begin{bmatrix} d\ln A \\ d\ln\psi_c \end{bmatrix} = \frac{\mathbb{E}_1(\pi)}{\mathbb{E}_1(\ell)} \begin{bmatrix} 1-f_x & f_x \\ 1-f_x & f_x-\delta \end{bmatrix} \begin{bmatrix} d\ln(F_e/L) \\ d\ln(F/L) \end{bmatrix}$$

where

$$\frac{\mathbb{E}_1(\pi)}{\mathbb{E}_1(\ell)} = \frac{1}{\mathbb{E}_\pi(\sigma)-1} = \{\mathbb{E}_r[\mu^{-1}]\}^{-1} - 1 = \mathbb{E}_\ell(\mu) - 1 > 0;$$

The average profit/the average labor cost ratio among the active firms

$$f_x \equiv \frac{FG(\psi_c)}{F_e + FG(\psi_c)} = \frac{\pi(\psi_c/A)}{\mathbb{E}_1(\pi)} < 1;$$

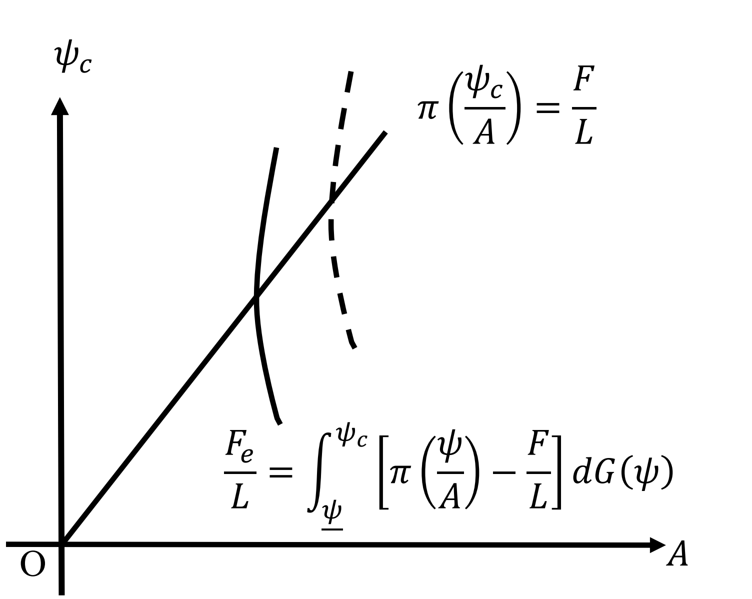
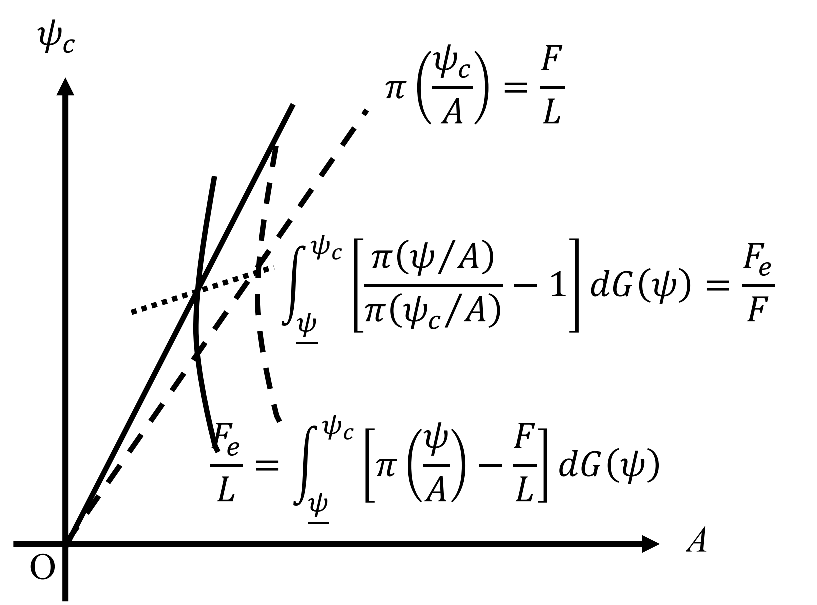
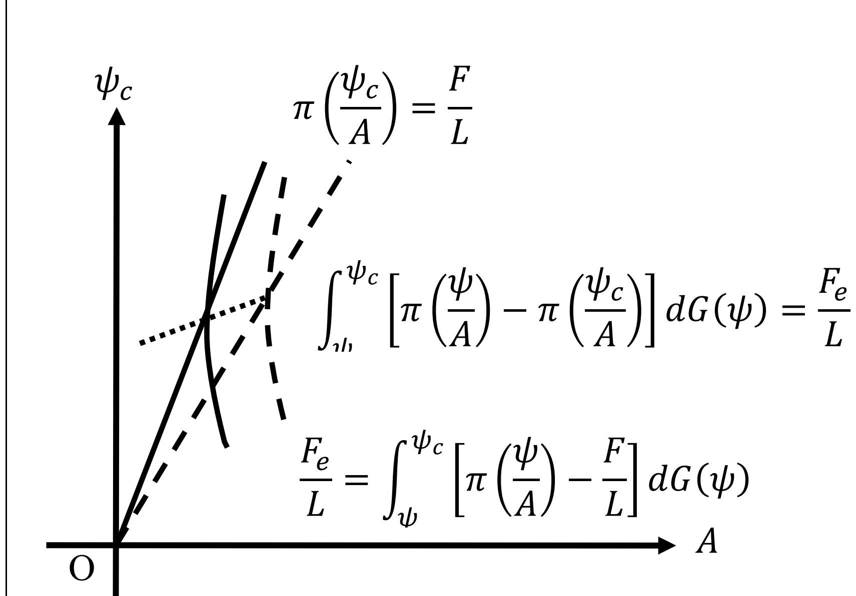
The share of the overhead in the total expected fixed cost = to the profit of the cut-off firm relative to the average profit among the active firms

$$\delta \equiv \frac{\mathbb{E}_\pi(\sigma)-1}{\sigma(\psi_c/A)-1} = \frac{\pi(\psi_c/A)}{\ell(\psi_c/A)}\frac{\mathbb{E}_1(\ell)}{\mathbb{E}_1(\pi)} \equiv f_x \frac{\mathbb{E}_1(\ell)}{\ell(\psi_c/A)} > 0.$$

The profit/labor cost ratio of the cut-off firm to the average profit/average labor cost ratio among the active firms.

Corollary of Proposition 6

	A	ψ_c/A	ψ_c
F_e	$\frac{dA}{dF_e} > 0$	$\frac{d(\psi_c/A)}{dF_e} = 0$	$\frac{d\psi_c}{dF_e} > 0$
L	$\frac{dA}{dL} < 0$	$\frac{d(\psi_c/A)}{dL} > 0$	$\frac{d\psi_c}{dL} < 0 \Leftrightarrow \mathbb{E}_\pi(\sigma) < \sigma\left(\frac{\psi_c}{A}\right)$, which holds globally if $\sigma'(\cdot) > 0$, i.e., under A2
F	$\frac{dA}{dF} > 0$	$\frac{d(\psi_c/A)}{dF} < 0$	$\frac{d\psi_c}{dF} > 0 \Leftrightarrow \mathbb{E}_1(\ell) < \ell\left(\frac{\psi_c}{A}\right)$, which holds globally if $\ell'(\cdot) > 0$

$F_e \downarrow$	$L \uparrow$ under A2 $\Leftrightarrow \mathbb{E}_\pi(\sigma) < \sigma(\psi_c/A)$	$F \downarrow$ under $\mathbb{E}_1(\ell) < \ell(\psi_c/A)$
		

Market Size Effect on Profit, $\Pi_\psi \equiv \pi(\psi/A)L$ and Revenue, $R_\psi \equiv r(\psi/A)L$ (Proposition 7)

7a: Under **A2,** there exists a unique $\psi_0 \in (\underline{\psi}, \psi_c)$ such that $\sigma\left(\frac{\psi_0}{A}\right) = \mathbb{E}_\pi(\sigma)$ with

$$\frac{d \ln \Pi_\psi}{d \ln L} > 0 \Leftrightarrow \sigma\left(\frac{\psi}{A}\right) < \mathbb{E}_\pi(\sigma) \text{ for } \psi \in \left(\underline{\psi}, \psi_0\right),$$

and

$$\frac{d \ln \Pi_\psi}{d \ln L} < 0 \Leftrightarrow \sigma\left(\frac{\psi}{A}\right) > \mathbb{E}_\pi(\sigma) \text{ for } \psi \in (\psi_0, \psi_c).$$

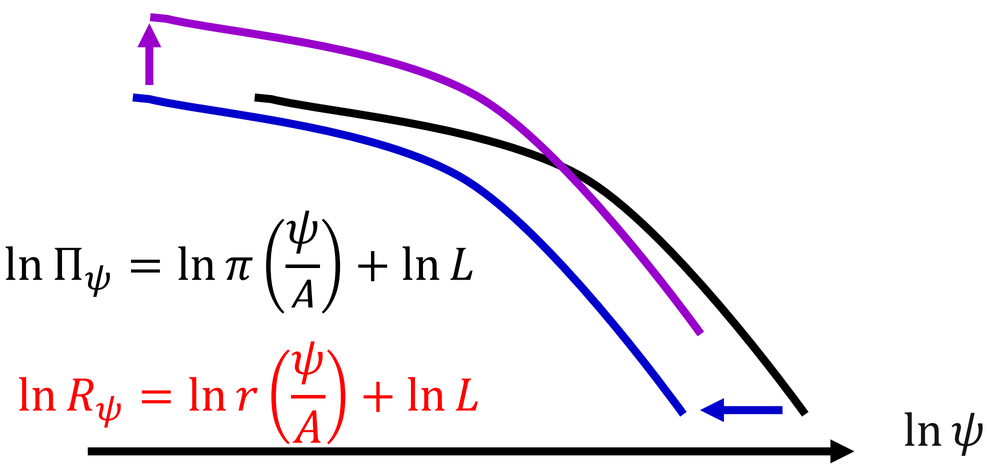
7b: Under **A2 and the weak A3,** there exists $\psi_1 > \psi_0$, such that

$$\frac{d \ln R_\psi}{d \ln L} > 0 \text{ for } \psi \in \left(\underline{\psi}, \psi_1\right).$$

Furthermore, $\psi_1 \in (\psi_0, \psi_c)$ and

$$\frac{d \ln R_\psi}{d \ln L} < 0 \text{ for } \psi \in (\psi_1, \psi_c),$$

for a sufficiently small F.



In short, more productive firms expand in absolute terms, while less productive firms shrink.

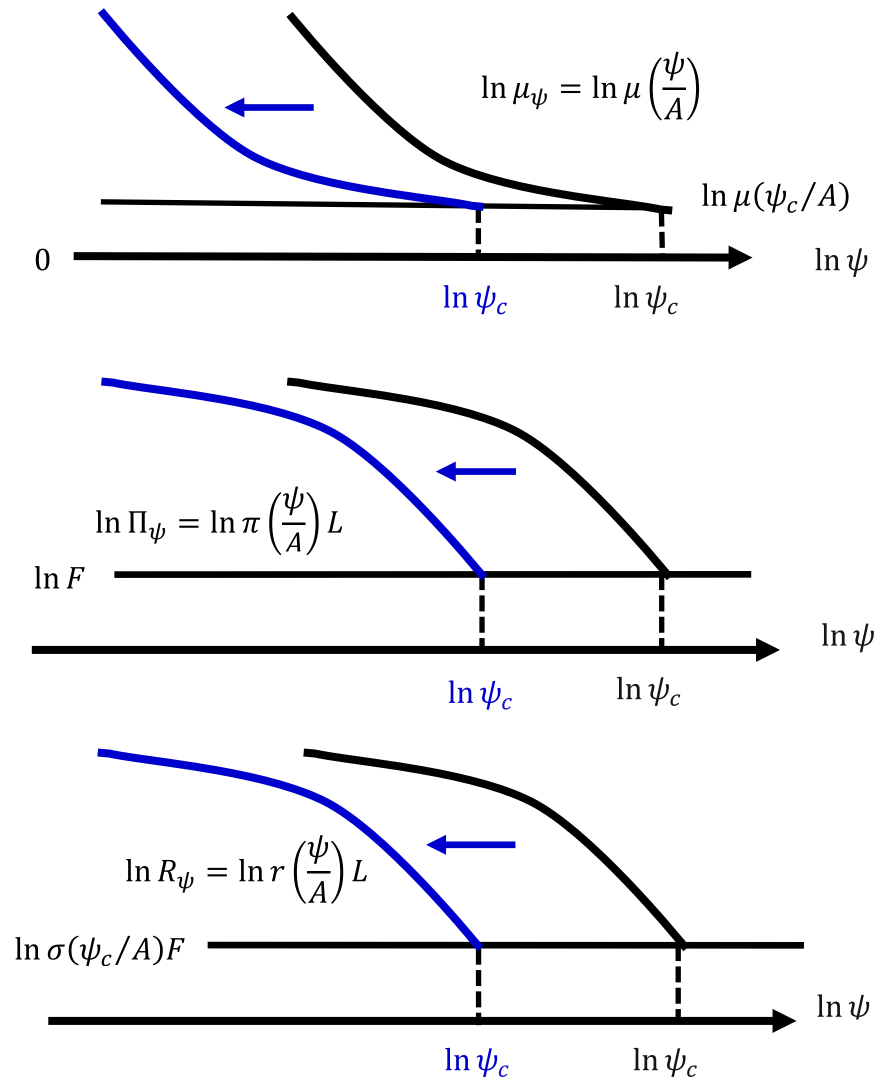
By putting together the main implications of Propositions 2, 3, 6, and 7

$F_e \downarrow$ under A2 and the weak A3

$A \downarrow$, $\psi_c \downarrow$ with ψ_c/A unchanged

The cutoff firms before the change and the cutoff firms after the change have

- the same markup rate $\mu(\psi_c/A)$
- the same profit $\pi(\psi_c/A)L = F$
- the same revenue, $r(\psi_c/A)L$



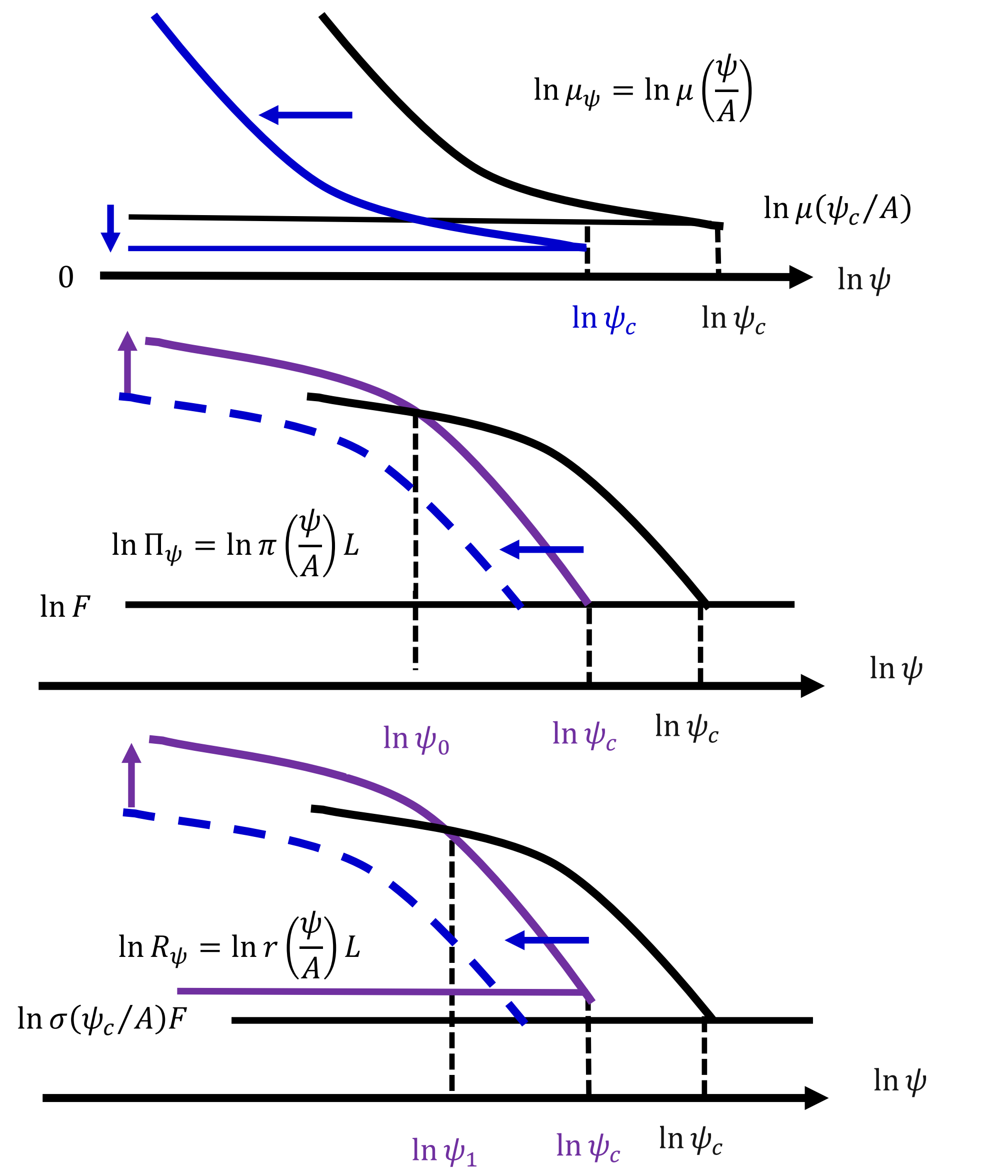
$L \uparrow$ under A2 and the weak A3

$A \downarrow$, $\psi_c \downarrow$ with $\psi_c/A \uparrow$ and $\sigma(\psi_c/A) \uparrow$

Compared to the cutoff firms before the change, the cutoff firms after the change have

- a lower markup rate, $\mu(\psi_c/A) \downarrow$
- the same profit, $\pi(\psi_c/A)L = F$.
- a higher revenue, $r(\psi_c/A)L = \sigma(\psi_c/A)F \uparrow$

Profits up (down) for firms with $\psi < (>)\psi_0$;
Revenues up (down) for firms with $\psi < (>)\psi_1$ for a sufficiently small F.

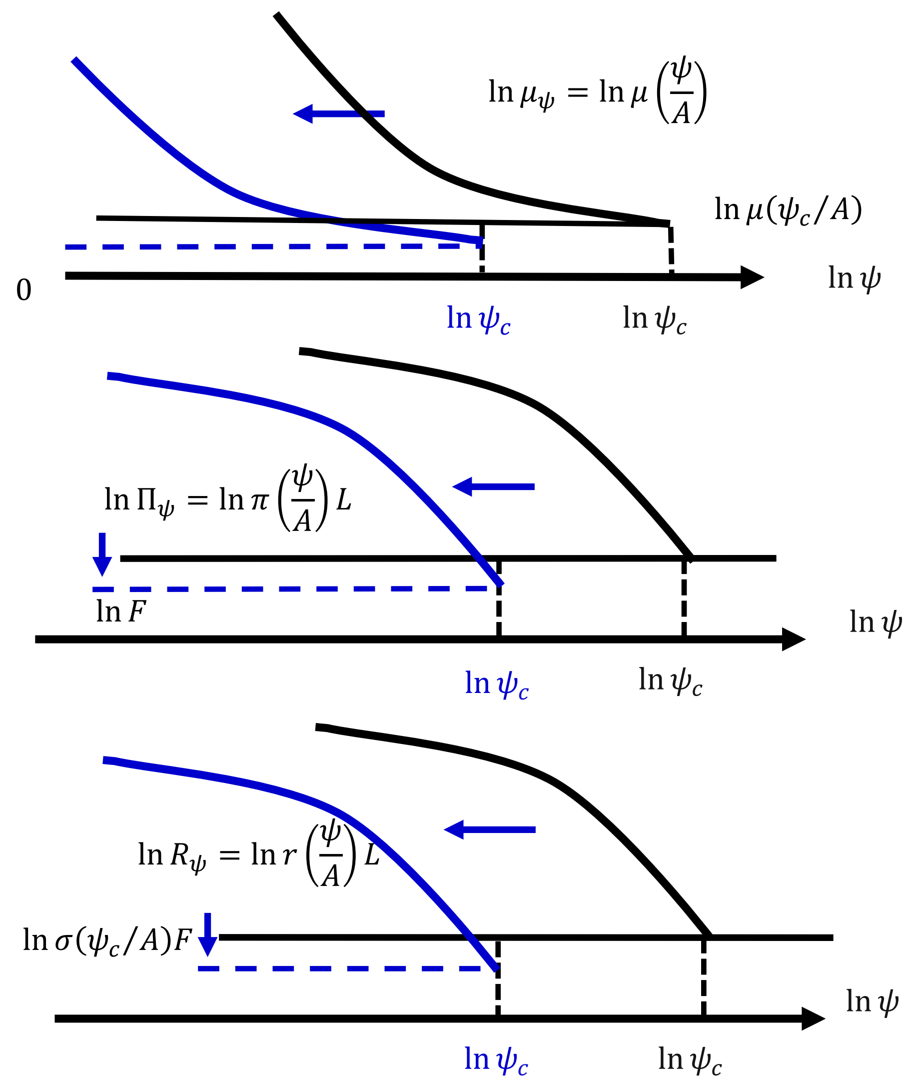


$F \downarrow$ under A2 and the weak A3 with $\ell'(\cdot) > 0$

$A \downarrow,\ \psi_c \downarrow$ with $\psi_c/A \uparrow$ and $\sigma(\psi_c/A) \uparrow$

Compared to the cutoff firms before the change, the cutoff firms after the change have

- a lower markup rate, $\mu(\psi_c/A) \downarrow$
- a lower profit, $\pi(\psi_c/A)L = F \downarrow$.
- a lower revenue, $r(\psi_c/A)L = \sigma(\psi_c/A)F \downarrow$.



The Composition Effect: Average Markup and Pass-Through Rates

- Under A2, $A\downarrow$ causes $\mu(\psi/A)\downarrow$ for each ψ, but distribution shifts toward low-ψ firms with higher $\mu(\psi/A)$.
- Under strong A3, $A\downarrow$ causes $\rho(\psi/A)\uparrow$ for each ψ, but distribution shifts toward low-ψ firms with lower $\rho(\psi/A)$.

Proposition 8: *Assume that $\mathcal{E}_g'(\cdot)$ does not change its sign and $\underline{\psi}=0$. Consider a shock to F_e, L, and/or F, which affects competitive pressures, i.e., $dA\neq 0$. Then, the response of any weighted generalized mean of any monotone function, $f(\psi/A)>0$, defined by*

$$I\equiv\mathcal{M}^{-1}\left(\mathbb{E}_w\big(\mathcal{M}(f)\big)\right)$$

with a monotone transformation $\mathcal{M}:\mathbb{R}_+\to\mathbb{R}$ and a weighting function, $w(\psi/A)>0$, satisfies:

	$f'(\cdot)>0$	$f'(\cdot)=0$	$f'(\cdot)<0$
$\mathcal{E}_g'(\cdot)>0$	$\frac{d\ln(\psi_c/A)}{d\ln A}\geq 0\Rightarrow\frac{d\ln I}{d\ln A}>0$	$\frac{d\ln I}{d\ln A}=0$	$\frac{d\ln(\psi_c/A)}{d\ln A}\geq 0\Rightarrow\frac{d\ln I}{d\ln A}<0$
$\mathcal{E}_g'(\cdot)=0$ (Pareto)	$\frac{d\ln(\psi_c/A)}{d\ln A}\gtreqless 0\Leftrightarrow\frac{d\ln I}{d\ln A}\gtreqless 0$	$\frac{d\ln I}{d\ln A}=0$	$\frac{d\ln(\psi_c/A)}{d\ln A}\gtreqless 0\Leftrightarrow\frac{d\ln I}{d\ln A}\lesseqgtr 0$
$\mathcal{E}_g'(\cdot)<0$	$\frac{d\ln(\psi_c/A)}{d\ln A}\leq 0\Rightarrow\frac{d\ln I}{d\ln A}<0$	$\frac{d\ln I}{d\ln A}=0$	$\frac{d\ln(\psi_c/A)}{d\ln A}\leq 0\Rightarrow\frac{d\ln I}{d\ln A}>0$

Moreover, if $\mathcal{E}_g'(\cdot)=\frac{d\ln(\psi_c/A)}{d\ln A}=0$, $d\ln I/d\ln A=0$ for any $f(\psi/A)$, monotonic or not. Furthermore, $\mathcal{E}_g'(\cdot)$ can be replaced with $\mathcal{E}_G'(\cdot)$ in all the above statements for $w(\psi/A)=1$, i.e., the unweighted averages.

The arithmetic, $I=\big(\mathbb{E}_w(f)\big)$, geometric, $I=\exp[\mathbb{E}_w(\ln f)]$, harmonic, $I=\big(\mathbb{E}_w(f^{-1})\big)^{-1}$, means are special cases.
The weight function, $w(\psi/A)$, can be profit, revenue, and employment.

Corollary 1 of Proposition 8

a) Entry Cost: $f'(\cdot)\mathcal{E}_g'(\cdot) \gtreqless 0 \Leftrightarrow \frac{d\ln I}{d\ln F_e} = \frac{d\ln I}{d\ln A}\frac{d\ln A}{d\ln F_e} \gtreqless 0.$

b) Market Size: *If* $\mathcal{E}_g'(\cdot) \leq 0$, *then,* $f'(\cdot) \gtreqless 0 \Rightarrow \frac{d\ln I}{d\ln L} = \frac{d\ln I}{d\ln A}\frac{d\ln A}{d\ln L} \gtreqless 0.$

c) Overhead Cost: *If* $\mathcal{E}_g'(\cdot) \leq 0$, *then,* $f'(\cdot) \gtreqless 0 \Rightarrow \frac{d\ln I}{d\ln F} = \frac{d\ln I}{d\ln A}\frac{d\ln A}{d\ln F} \lesseqgtr 0.$

Furthermore, $\mathcal{E}_g'(\cdot)$ *can be replaced with* $\mathcal{E}_G'(\cdot)$ *for* $w(\psi/A) = 1$, *i.e., the unweighted averages.*

For the entry cost, $\frac{d\ln(\psi_c/A)}{d\ln A} = 0.$

- $\mathcal{E}_g'(\cdot) > 0$; sufficient & necessary for the composition effect to dominate:
    - The average markup & pass-through rates move in the *opposite* direction from the firm-level rates
- $\mathcal{E}_g'(\cdot) = 0$ (Pareto); a knife-edge. $A \downarrow$ → no change in average markup and pass-through.
- $\mathcal{E}_g'(\cdot) < 0$; sufficient & necessary for the procompetitive effect to dominate:
  The average markup & pass-through rates move in the *same* direction from the firm-level rates

For market size and the overhead cost, $\frac{d\ln(\psi_c/A)}{d\ln A} < 0$

- $\mathcal{E}_g'(\cdot) > 0$; necessary for the composition effect to dominate:
- $\mathcal{E}_g'(\cdot) \leq 0$; sufficient for the procompetitive effect to dominate:

The Composition Effect: Impact on P/A

$$\ln\left(\frac{A}{cP}\right) = \mathbb{E}_r[\Phi \circ Z]$$

$$\zeta'(\cdot) \gtreqless 0 \;\Longrightarrow \Phi'(\cdot) \lesseqgtr 0 \Leftrightarrow \Phi \circ Z'(\cdot) \lesseqgtr 0$$

Corollary 2 of Proposition 8: *Assume $\underline{\psi} = 0$, and neither $\zeta'(\cdot)$ nor $\mathcal{E}_g'(\cdot)$ change the signs. Consider a shock to F_e, L, and/or F, which affects competitive pressures, i.e., $dA \neq 0$. Then, the response of P/A satisfies:*

	$\zeta'(\cdot) > 0$ (A2)	$\zeta'(\cdot) = 0$ (CES)	$\zeta'(\cdot) < 0$
$\mathcal{E}_g'(\cdot) > 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \geq 0 \Longrightarrow \frac{d\ln(P/A)}{d\ln A} > 0$	$\frac{d\ln(P/A)}{d\ln A} = 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \geq 0 \Longrightarrow \frac{d\ln(P/A)}{d\ln A} < 0$
$\mathcal{E}_g'(\cdot) = 0$ (Pareto)	$\frac{d\ln(\psi_c/A)}{d\ln A} \gtreqless 0 \Leftrightarrow \frac{d\ln(P/A)}{d\ln A} \gtreqless 0$	$\frac{d\ln(P/A)}{d\ln A} = 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \gtreqless 0 \Leftrightarrow \frac{d\ln(P/A)}{d\ln A} \lesseqgtr 0$
$\mathcal{E}_g'(\cdot) < 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \leq 0 \Longrightarrow \frac{d\ln(P/A)}{d\ln A} < 0$	$\frac{d\ln(P/A)}{d\ln A} = 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \leq 0 \Longrightarrow \frac{d\ln(P/A)}{d\ln A} > 0$

Comparative Statics on M& $MG(\psi_c)$

Proposition 9: *Assume that $\mathcal{E}_G'(\cdot)$ does not change its sign and $\underline{\psi} = 0$. Consider a shock to F_e, F, and/or L, which affects competitive pressures, i.e., $dA \neq 0$. Then, the response of the mass of active firms, $MG(\psi_c)$, is as follows*:

$$If\ \mathcal{E}_G'(\cdot) > 0, \qquad \frac{d\ln(\psi_c/A)}{d\ln A} \geq 0 \Rightarrow \frac{d\ln[MG(\psi_c)]}{d\ln A} > 0;$$
$$If\ \mathcal{E}_G'(\cdot) = 0, \qquad \frac{d\ln(\psi_c/A)}{d\ln A} \gtreqless 0 \Leftrightarrow \frac{d\ln[MG(\psi_c)]}{d\ln A} \gtreqless 0;$$
$$If\ \mathcal{E}_G'(\cdot) < 0, \qquad \frac{d\ln(\psi_c/A)}{d\ln A} \leq 0 \Rightarrow \frac{d\ln[MG(\psi_c)]}{d\ln A} < 0.$$

Corollary 1 of Proposition 9

a) Entry Cost: $\mathcal{E}_G'(\cdot) \gtreqless 0 \Leftrightarrow \frac{d\ln[MG(\psi_c)]}{d\ln F_e} = \frac{d\ln[MG(\psi_c)]}{d\ln A}\frac{d\ln A}{d\ln F_e} \gtreqless 0.$

b) Market Size: $\mathcal{E}_G'(\cdot) \leq 0 \Rightarrow \frac{d\ln[MG(\psi_c)]}{d\ln L} = \frac{d\ln[MG(\psi_c)]}{d\ln A}\frac{d\ln A}{d\ln L} > 0.$

c) Overhead Cost: $\mathcal{E}_G'(\cdot) \leq 0 \Rightarrow \frac{d\ln[MG(\psi_c)]}{d\ln F} = \frac{d\ln[MG(\psi_c)]}{d\ln A}\frac{d\ln A}{d\ln F} < 0.$

For a decline in the entry cost,
$\mathcal{E}_g'(\cdot) > 0$ *sufficient & necessary* for $MG(\psi_c)\downarrow$; $\mathcal{E}_g'(\cdot) = 0$, no effect; $\mathcal{E}_g'(\cdot) < 0$; *sufficient & necessary* for $MG(\psi_c)\uparrow$
For market size and the overhead cost
$\mathcal{E}_g'(\cdot) > 0$ *necessary* for $MG(\psi_c)\downarrow$; $\mathcal{E}_g'(\cdot) \leq 0$ *sufficient* for $MG(\psi_c)\uparrow$

Impact of Competitive Pressures on Unit Cost/TFP

By combining Corollary 2 of Proposition 8 and Corollary 1 of Proposition,

Corollary 2 of Proposition 9: *Assume* $\underline{\psi} = 0$, *and neither* $\zeta'(\cdot)$ *nor* $\mathcal{E}_g'(\cdot)$ *change the signs. Consider a shock to* F_e, L, *and/or* F, *which affects competitive pressures, i.e.,* $dA \neq 0$. *Then, the response of* P *satisfies:*

	$\zeta'(\cdot) > 0$ (A2)	$\zeta'(\cdot) = 0$ (CES)	$\zeta'(\cdot) < 0$
$\mathcal{E}_g'(\cdot) > 0$	$\frac{d \ln P}{d \ln A} > 1\ for\ F_e$	$\frac{d \ln P}{d \ln A} = 1$	?
$\mathcal{E}_g'(\cdot) = 0$ (Pareto)	$\frac{d \ln P}{d \ln A} = 1\ for\ F_e$ $0 < \frac{d \ln P}{d \ln A} < 1\ for\ F\ or\ L;$	$\frac{d \ln P}{d \ln A} = 1$	$\frac{d \ln P}{d \ln A} = 1\ for\ F_e$ $\frac{d \ln P}{d \ln A} > 1\ for\ F\ or\ L$
$\mathcal{E}_g'(\cdot) < 0$	$0 < \frac{d \ln P}{d \ln A} < 1$	$\frac{d \ln P}{d \ln A} = 1$	$\frac{d \ln P}{d \ln A} > 1$

Limit Case of $F \to 0$ with $\bar{z} < \infty$

Cutoff Rule:	$\pi\left(\frac{\psi_c}{A}\right) = 0 \Leftrightarrow \frac{\psi_c}{A} = \bar{z} = \pi^{-1}(0)$
Free Entry Condition:	$\frac{F_e}{L} = \int_{\underline{\psi}}^{\psi_c} \pi\left(\bar{z}\frac{\psi}{\psi_c}\right) dG(\psi) = \int_{\underline{\psi}}^{\bar{z}A} \pi\left(\frac{\psi}{A}\right) dG(\psi).$

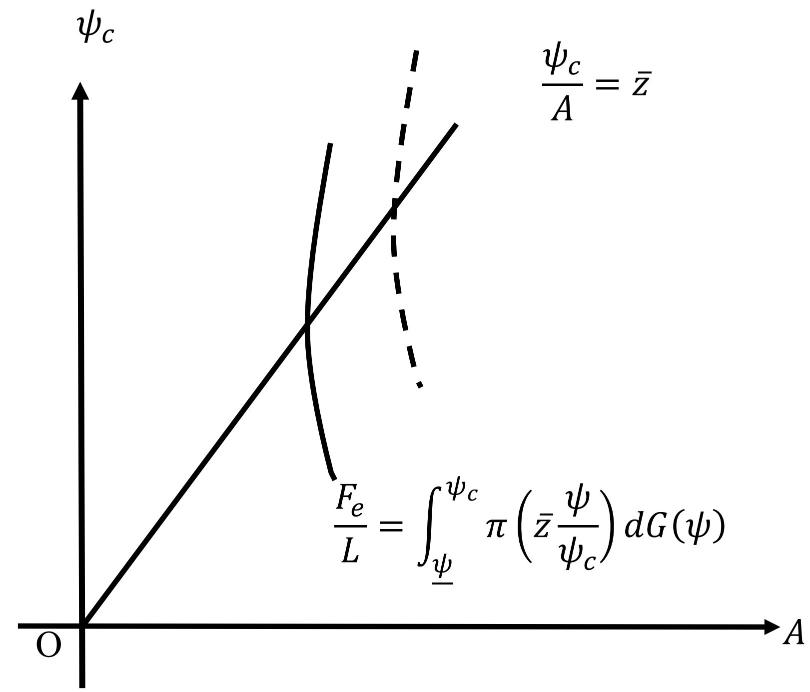
A and ψ_c: uniquely determined as C^2 functions of F_e/L with the interior solution, $0 < G(\psi_c) < 1$ for

$$0 < \frac{F_e}{L} < \int_{\underline{\psi}}^{\bar{\psi}} \pi\left(\bar{z}\frac{\psi}{\bar{\psi}}\right) dG(\psi).$$

$$\frac{d\psi_c}{\psi_c} = \frac{dA}{A} = \frac{1}{\mathbb{E}_\pi(\sigma) - 1} \frac{d(F_e/L)}{F_e/L}$$

$$\frac{dM}{d(F_e/L)} < 0; \quad \mathcal{E}_G'(\psi) \lesseqgtr 0 \Leftrightarrow \frac{d[MG(\psi_c)]}{d(F_e/L)} \lesseqgtr 0$$

$L \uparrow$ is isomorphic to $F_e \downarrow$.



$F_e/L \downarrow$ for $F \to 0$ with $\bar{z} < \infty$ under A2 and the weak A3

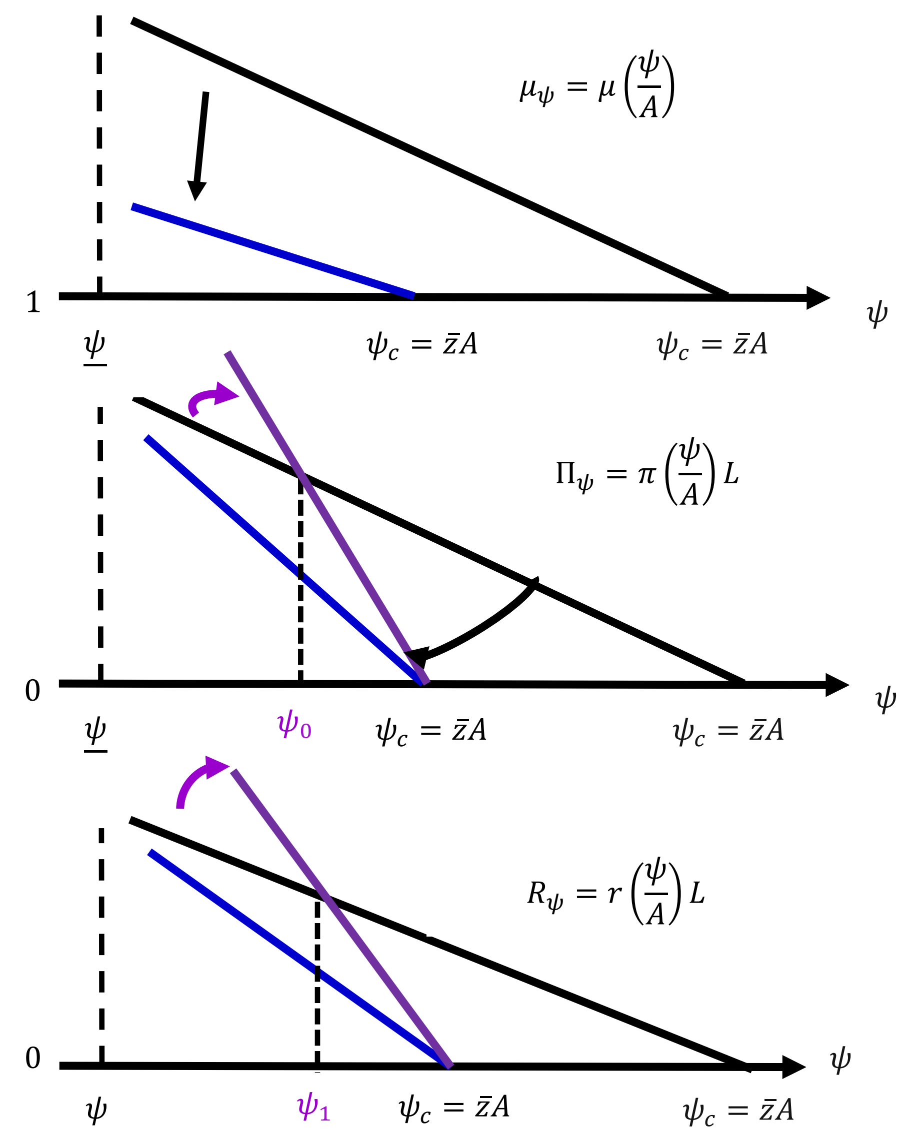
$A \downarrow, \psi_c \downarrow$ with $\psi_c/A = \bar{z}$ unchanged.

The cutoff firms always (i.e., both before and after the change) have

- $\mu(\psi_c/A) = 1$
- $\pi(\psi_c/A)L = 0.$
- $r(\psi_c/A)L = 0.$

Profits up (down) for firms with $\psi < (>)\psi_0$;
Revenues up (down) for firms with $\psi < (>)\psi_1$.

In the middle and the lower panels,
Blue : the effects of $F_e/L \downarrow$ due to $F_e \downarrow$
Purple: the effects of $F_e/L \downarrow$ due to $L \uparrow$



Sorting of Heterogeneous Firms: A Multi-Market Setting

A Multi-Market Extension: J markets, $j = 1,2, \dots, J$, with market size L_j.

Possible Interpretations

- Identical Households with Cobb-Douglas preferences, $\sum_{j=1}^{J} \beta_j \ln X_j$ with $\sum_{j=1}^{J} \beta_j = 1$. Then, $L_j = \beta_j L$.
- J types of consumers, with L_j equal to the total income of type-j consumers. "Types" can be their "tastes" or "locations", etc.

Assume

- Market size is the only exogenous source of heterogeneity across markets: Index them as $L_1 > L_2 >, \dots, > L_J > 0$.
- Labor is fully mobile, equalizing the wage across the markets. We continue to use it as the *numeraire*.
- Firm's marginal cost, ψ, is independent of the market it chooses.
  - Each firm pays $F_e > 0$ to draw its marginal cost $\psi \sim G(\psi)$.
  - Knowing its ψ, each firm decides which market to enter and produce with an overhead cost, $F > 0$, or exit without producing.
  - Firms sell their products at the profit-maximizing prices in the market they enter.

Equilibrium Condition:

$$F_e = \int_{\underline{\psi}}^{\overline{\psi}} \max\{\Pi_\psi - F, 0\}\, dG(\psi) = \int_{\underline{\psi}}^{\overline{\psi}} \max\left\{\max_{1\le j\le J}\{\Pi_{j\psi}\} - F, 0\right\} dG(\psi)$$

$$\text{where} \quad \Pi_{j\psi} \equiv \frac{s\left(Z(\psi/A_j)\right)}{\zeta\left(Z(\psi/A_j)\right)} L_j \equiv \frac{r(\psi/A_j)}{\sigma(\psi/A_j)} L_j = \pi\left(\frac{\psi}{A_j}\right) L_j$$

Proposition 10: Equilibrium Characterization under A2

Larger markets are more competitive:

$$0 < A_1 < A_2 < \cdots < A_J < \infty, \text{where } M \int_{\psi_{j-1}}^{\psi_j} r\left(\frac{\psi}{A_j}\right) dG(\psi) = 1.$$

Note: Because $\pi(\cdot)$ is strictly decreasing, this implies $\pi(\psi/A_1) < \pi(\psi/A_2) < \cdots < \pi(\psi/A_J)$ for all ψ.

More productive firms self-select into larger markets (Positive Assortative Matching)

Firms with $\psi \in (\psi_{j-1}, \psi_j)$ enter market-j and those with $\psi \in (\psi_J, \infty)$ do not enter any market, where

$$0 \leq \underline{\psi} = \psi_0 < \psi_1 < \psi_2 < \cdots < \psi_J < \overline{\psi} \leq \infty \quad \text{where } \frac{\pi(\psi_j/A_j)L_j}{\pi(\psi_j/A_{j+1})L_{j+1}} = 1 \text{ for } 1 \leq j \leq J-1; \quad \pi\left(\frac{\psi_J}{A_J}\right)L_J \equiv F$$

Note: ψ_j-firms are indifferent btw entering Market-j & entering Market-$(j+1)$.

Free Entry Condition:

$$\sum_{j=1}^{J} \int_{\psi_{j-1}}^{\psi_j} \left\{\pi\left(\frac{\psi}{A_j}\right) L_j - F\right\} dG(\psi) = F_e$$

Mass of Firms in Market-*j*:

$$M[G(\psi_j) - G(\psi_{j-1})] > 0$$

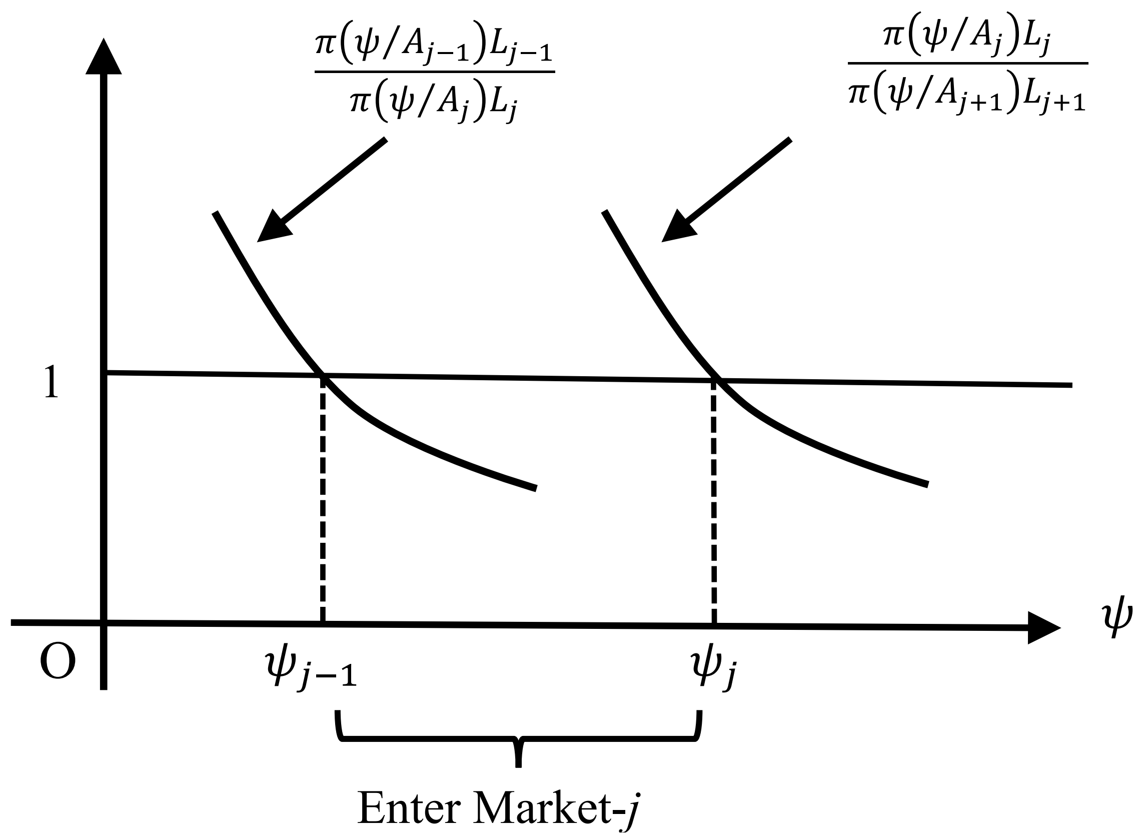
Logic Behind Sorting

$L_j > L_{j+1} \Longrightarrow A_j < A_{j+1}$. Otherwise, no firm would enter $j+1$.

$\Longrightarrow \frac{\pi(\psi/A_j)}{\pi(\psi/A_{j+1})}$ strictly decreasing in ψ

due to strict log-supermodularity of $\pi(\psi/A)$ under **A2**

$$\Longrightarrow \left[\frac{\Pi_{j\psi}}{\Pi_{(j+1)\psi}} = \frac{\pi(\psi/A_j)L_j}{\pi(\psi/A_{j+1})L_{j+1}} \gtreqless 1 \Leftrightarrow \psi \lesseqgtr \psi_j\right]$$



Under CES, $\frac{\pi(\psi/A_j)L_j}{\pi(\psi/A_{j+1})L_{j+1}}$ is independent of ψ.

$\Longrightarrow \frac{\Pi_{j\psi}}{\Pi_{(j+1)\psi}} = \frac{\pi(\psi/A_j)L_j}{\pi(\psi/A_{j+1})L_{j+1}} = 1$ in equilibrium.

$\Longrightarrow$ Firms indifferent across all markets.

$\Longrightarrow$ Distribution of firms across markets is indeterminate.

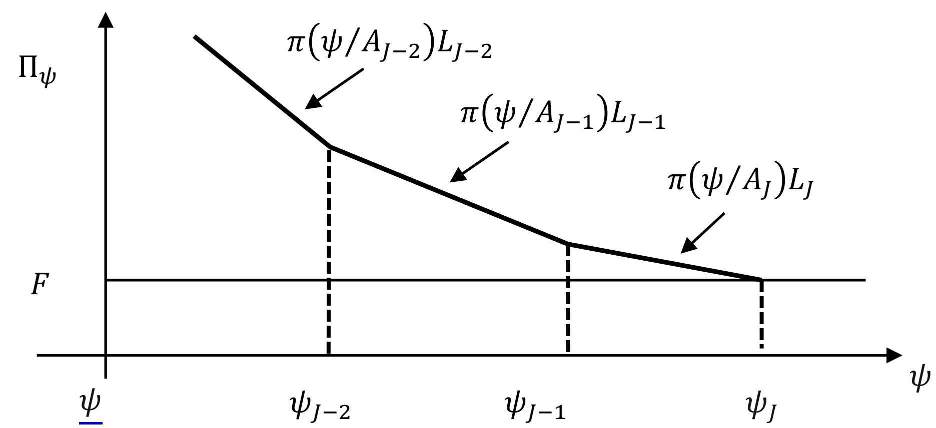
Our mechanism generates sorting through competitive pressures. As such,

- complementary to agglomeration-economies-based mechanisms offered by Gaubert (2018) and Davis-Dingel (2019)
- justifies the equilibrium selection criterion used by Baldwin-Okubo (2006), which use CES, as a limit argument.

Cross-Sectional, Cross-Market Implications:

Profits: Under A2

$$L_j > L_{j+1} \Rightarrow A_j < A_{j+1} \Rightarrow \left[\frac{\pi(\psi/A_j)L_j}{\pi(\psi/A_{j+1})L_{j+1}} \gtreqless 1 \Leftrightarrow \psi \lesseqgtr \psi_j\right]$$

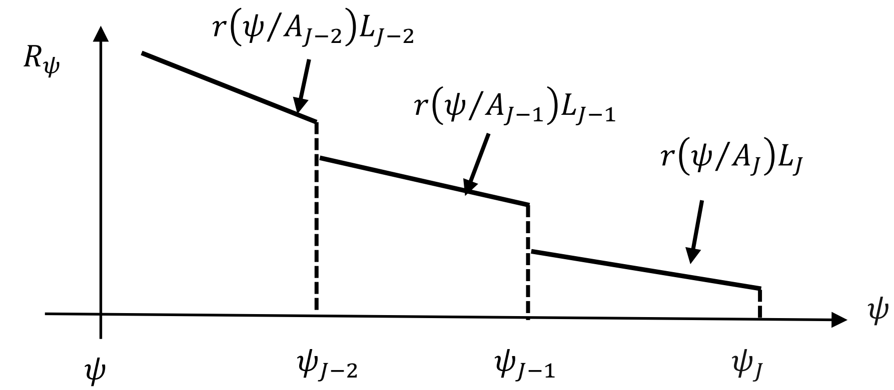


$\Pi_\psi = \max_j \left\{\pi\left(\frac{\psi}{A_j}\right)L_j\right\}$, the upper-envelope of $\pi(\psi/A_j)L_j$, is continuous and decreasing in ψ, with the kinks at ψ_j.

Continuous, since the lower markup rate in Market-j cancels out its larger market size, keeping ψ_j-firms indiffierent btw Market-j & Market-$(j+1)$.

Revenues: Under A2

$$\frac{r(\psi_j/A_j)L_j}{r(\psi_j/A_{j+1})L_{j+1}} = \frac{\sigma(\psi_j/A_j)\pi(\psi_j/A_j)L_j}{\sigma(\psi_j/A_{j+1})\pi(\psi_j/A_{j+1})L_{j+1}} = \frac{\sigma(\psi_j/A_j)}{\sigma(\psi_j/A_{j+1})} > 1$$

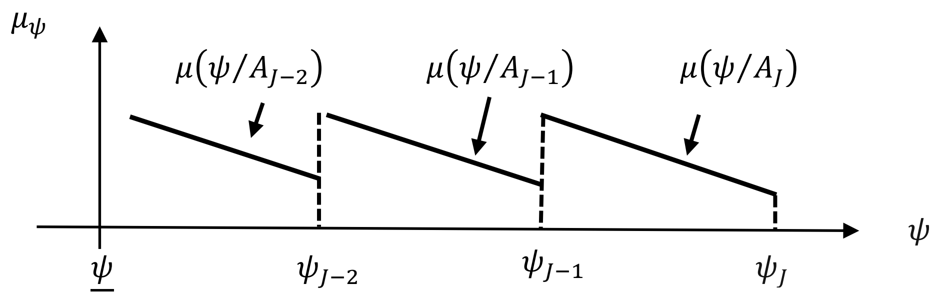


R_ψ: continuously decreasing in ψ within each market; jumps down at ψ_j.
With the markup rate lower in Market-j, ψ_j-firms need to earn higher revenue to keep them indiffierent btw Market-j & and Market-$(j+1)$.

Markup Rates: Under A2

$$L_j > L_{j+1} \Rightarrow A_j < A_{j+1} \Rightarrow \sigma\left(\frac{\psi_j}{A_j}\right) > \sigma\left(\frac{\psi_j}{A_{j+1}}\right) \Leftrightarrow \mu\left(\frac{\psi_j}{A_j}\right) < \mu\left(\frac{\psi_j}{A_{j+1}}\right)$$

μ_ψ: continuously decreasing in ψ within each market but jumps up at ψ_j.

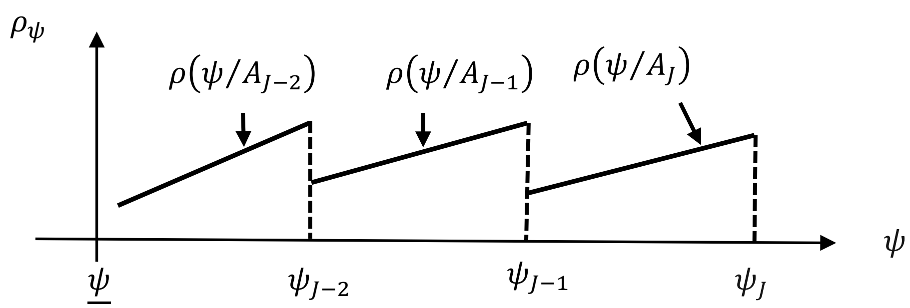


- The average markup rates may be *higher* in larger (and hence more competitive) markets.
- The average markup rates in all markets may *go up*, even if all markets become more competitive ($A_j \downarrow$).

Pass-Through Rates: Under A2 and the strong A3

$$L_j > L_{j+1} \Rightarrow A_j < A_{j+1} \Rightarrow \rho\left(\frac{\psi_j}{A_j}\right) > \rho\left(\frac{\psi_j}{A_{j+1}}\right)$$

ρ_ψ: continuously increasing in ψ within each market but jumps down at ψ_j.



- The average pass-through rates may be *lower* in larger (and hence more competitive) markets.
- The average pass-through rates in all markets go *down* even if all markets become more competitive ($A_j \downarrow$).

Average Markup and Pass-Through Rates in a Multi-Market Model: The Composition Effect

Proposition 11a: *Suppose A2 and* $G(\psi) = \left(\psi/\overline{\psi}\right)^{\kappa}$. *There exists a sequence,* $L_1 > L_2 > \cdots > L_J > 0$, *such that, in equilibrium, any weighted generalized mean of* $f\left(\psi/A_j\right)$ *across firms operating at market-*j *are increasing (decreasing) in* j *even though* $f(\cdot)$ *is increasing (decreasing) and hence* $f\left(\psi/A_j\right)$ *is decreasing (increasing) in* j.

Corollary of Proposition 11a: An example with $G(\psi) = \left(\psi/\overline{\psi}\right)^{\kappa}$, such that the average markup rates are *higher* (and the average pass-through rates are *lower* under Strong A3) in larger markets.

Proposition 11b: *Suppose A2 and* $G(\psi) = \left(\psi/\overline{\psi}\right)^{\kappa}$. *Then, a change in* F_e *keeps*

i) the ratios $a_j \equiv \psi_{j-1}/\psi_j$ *and* $b_j \equiv \psi_j/A_j$

and

ii) any weighted generalized mean of $f\left(\psi/A_j\right)$ *across firms operating at market-*j, *for any weighting function* $w\left(\psi/A_j\right)$,

unchanged for all $j = 1,2,\ldots,J$.

Corollary of Proposition 11b: $F_e \downarrow$ and $G(\psi) = \left(\psi/\overline{\psi}\right)^{\kappa}$ offers a knife-edge case, where the average markup and pass-through rates of all markets remain unchanged.

A caution against testing A2/A3 by comparing the average markup & pass-through rates across space and time.

Appendices

Symmetric H.S.A. with Gross Substitutes: An Alternative (Equivalent) Definition

Market Share of ω depends *solely* on its own quantity normalized by the *common* quantity aggregator

$$s_\omega \equiv \frac{p_\omega x_\omega}{\mathbf{px}} = \frac{\partial \ln X(\mathbf{x})}{\partial \ln x_\omega} = s^*\left(\frac{x_\omega}{A^*(\mathbf{x})}\right), \qquad \text{Where} \quad \int_\Omega s^*\left(\frac{x_\omega}{A^*(\mathbf{x})}\right) d\omega \equiv 1.$$

- $s^*: \mathbb{R}_{++} \to \mathbb{R}_+$: **the market share function**, with $0 < \mathcal{E}_{s^*}(y_\omega) < 1$, where $y_\omega \equiv x_\omega / A^*$ is **the normalized quantity**
  - If $\bar{z} \equiv s^{*\prime}(0) = \lim_{y\to 0}[s^*(y)/y] < \infty$, $\bar{z}A(\mathbf{p})$ is the **choke price**.
- $A^* = A^*(\mathbf{x})$: the **common quantity aggregator** defined implicitly by **the adding up constraint** $\int_\Omega s^*(x_\omega/A^*) d\omega \equiv 1$. $A^*(\mathbf{x})$ linear homogenous in $\mathbf{x}$ for a fixed Ω. A larger Ω raises $A^*(\mathbf{x})$.

Two definitions equivalent with the one-to-one mapping, $s(z) \leftrightarrow s^*(y)$, defined by $s^* \equiv s(s^*/y)$ or $s \equiv s^*(s/z)$.

CES if $s^*(y) = \gamma^{1/\sigma} y^{1-1/\sigma}$; CoPaTh if $s^*(y) = \left[(\gamma)^{\frac{\rho-1}{\rho}} + (y\bar{z})^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$ with $\rho \in (0,1)$.

Production Function: $X(\mathbf{x}) = c^* A^*(\mathbf{x}) \exp\left\{\int_\Omega \left[\int_0^{x_\omega/A^*(\mathbf{x})} s^*(\xi)\frac{d\xi}{\xi}\right] d\omega\right\}$

Note: Our 2020 paper proved

$$\left[1 - \frac{d \ln s(z)}{d \ln z}\right]\left[1 - \frac{d \ln s^*(y)}{d \ln y}\right] = 1$$

Our 2017 paper proved that $X(\mathbf{x})$ is quasi-concave & that $A^*(\mathbf{x})/X(\mathbf{x}) = P(\mathbf{p})/A(\mathbf{p}) \neq c$ for any $c > 0$ unless CES

- ✓ $A^*(\mathbf{x})$, the measure of *competitive pressures*, fully captures *cross quantity effects* in the inverse demand system
- ✓ $X(\mathbf{x})$, the measure of output, captures the *output implications* of input changes

Labor Market Equilibrium: satisfied automatically from the Walras Law.

$$Labor\ Demand = M\left[F_e + \int_{\underline{\psi}}^{\psi_c}(x_\psi \psi + F)dG(\psi)\right] = M\left[F_e + FG(\psi_c) + \int_{\underline{\psi}}^{\psi_c} \ell\left(\frac{\psi}{A}\right) LdG(\psi)\right]$$

$$= LM\left[\int_{\underline{\psi}}^{\psi_c}\left[\pi\left(\frac{\psi}{A}\right) + \ell\left(\frac{\psi}{A}\right)\right] dG(\psi)\right] \qquad \text{(from the Free Entry Condition)}$$

$$= LM\int_{\underline{\psi}}^{\psi_c} r\left(\frac{\psi}{A}\right) dG(\psi) = L \qquad \text{(from the Adding Up Constraint)}$$

Three Parametric Families of H.S.A.

Generalized Translog:

$$s(z) = \gamma\left(1 - \frac{\sigma-1}{\eta}\ln\left(\frac{z}{\beta}\right)\right)^{\eta} = \gamma\left(-\frac{\sigma-1}{\eta}\ln\left(\frac{z}{\bar{z}}\right)\right)^{\eta};\ z < \bar{z} \equiv \beta e^{\frac{\eta}{\sigma-1}}$$

$$\Rightarrow \zeta(z) = 1 + \frac{\sigma-1}{1 - \frac{\sigma-1}{\eta}\ln\left(\frac{z}{\beta}\right)} = 1 - \frac{\eta}{\ln\left(\frac{z}{\bar{z}}\right)} > 1$$

$$\Rightarrow \eta z\zeta'(z) = [\zeta(z)-1]^2 \Rightarrow \frac{z\zeta'(z)}{[\zeta(z)-1]\zeta(z)} = \frac{1}{\eta}\left[1 - \frac{1}{\zeta(z)}\right] = \frac{1}{\eta - \ln\left(\frac{z}{\bar{z}}\right)}$$

satisfying **A2** but violating **A3.**

- CES is the limit case, as $\eta \to \infty$, while holding $\beta > 0$ and $\sigma > 1$ fixed, so that $\bar{z} \equiv \beta e^{\frac{\eta}{\sigma-1}} \to \infty$.
- Translog is the special case where $\eta = 1$.
- $z = Z\left(\frac{\psi}{A}\right)$ is given as the inverse of $\frac{\eta z}{\eta - \ln(z/\bar{z})} = \frac{\psi}{A}$;
- If $\eta \geq 1$, employment is globally decreasing in z;
- If $\eta < 1$, employment is hump-shaped with the peak, given by $\hat{z}/\bar{z} = \frac{\hat{\psi}}{(1-\eta)\bar{z}A} = \exp\left[-\frac{\eta^2}{1-\eta}\right] < 1$, decreasing in η.

Constant Pass-Through (CoPaTh): Matsuyama-Ushchev (2020b). For $0 < \rho < 1, \sigma > 1, \bar{z} \equiv \beta \left(\frac{\sigma}{\sigma-1}\right)^{\frac{\rho}{1-\rho}}$

$$s(z) = \gamma\sigma^{\frac{\rho}{1-\rho}}\left[1 - \left(\frac{z}{\bar{z}}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}} \Rightarrow 1 - \frac{1}{\zeta(z)} = \left(\frac{z}{\bar{z}}\right)^{\frac{1-\rho}{\rho}} < 1 \Rightarrow \mathcal{E}_{1-1/\zeta}(z) = -\mathcal{E}_{\zeta/(\zeta-1)}(z) = \frac{1-\rho}{\rho} > 0$$

satisfying **A2** and the weak form of **A3 (**but not the strong form**).** Then, for $\psi/A < \bar{z}$,

$$p_\psi = (\bar{z}A)^{1-\rho}(\psi)^\rho; \qquad Z\left(\frac{\psi}{A}\right) = (\bar{z})^{1-\rho}\left(\frac{\psi}{A}\right)^\rho;$$

$$\sigma\left(\frac{\psi}{A}\right) = \frac{1}{1-(\psi/\bar{z}A)^{1-\rho}}; \qquad \rho\left(\frac{\psi}{A}\right) = \rho$$

$$r\left(\frac{\psi}{A}\right) = \gamma\sigma^{\frac{\rho}{1-\rho}}\left[1 - \left(\frac{\psi}{\bar{z}A}\right)^{1-\rho}\right]^{\frac{\rho}{1-\rho}};\ \pi\left(\frac{\psi}{A}\right) = \gamma\sigma^{\frac{\rho}{1-\rho}}\left[1 - \left(\frac{\psi}{\bar{z}A}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}};\ \ell\left(\frac{\psi}{A}\right) = \gamma\sigma^{\frac{\rho}{1-\rho}}\left(\frac{\psi}{\bar{z}A}\right)^{1-\rho}\left[1 - \left(\frac{\psi}{\bar{z}A}\right)^{1-\rho}\right]^{\frac{\rho}{1-\rho}}$$

with

- a constant pass-through rate, $0 < \rho < 1$.
- Employment hump-shaped with $\hat{z}/\bar{z} = (1-\rho)^{\frac{\rho}{1-\rho}} > \hat{\psi}/\bar{z}A = (1-\rho)^{\frac{1}{1-\rho}}$, both decreasing in ρ.
- CES is the limit case, as $\rho \to 1$, while holding $\beta > 0$ and $\sigma > 1$ fixed, so that $\sigma(\psi/A) \to \sigma$; $\bar{z} \equiv \beta\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\rho}{1-\rho}} \to \infty$.

Power Elasticity of Markup Rate (Fréchet Inverse Markup Rate): For $\kappa \geq 0$ and $\lambda > 0$

$$s(z) = \exp\left[\int_{z_0}^{z} \frac{c}{c - \exp\left[-\frac{\kappa \bar{z}^{-\lambda}}{\lambda}\right]\exp\left[\frac{\kappa \xi^{-\lambda}}{\lambda}\right]}\frac{d\xi}{\xi}\right]$$

with either $\bar{z} = \infty$ and $c \leq 1$ or $\bar{z} < \infty$ and $c = 1$. Then,

$$1 - \frac{1}{\zeta(z)} = c\exp\left[\frac{\kappa \bar{z}^{-\lambda}}{\lambda}\right]\exp\left[-\frac{\kappa z^{-\lambda}}{\lambda}\right] < 1 \Rightarrow \mathcal{E}_{1-1/\zeta}(z) = -\mathcal{E}_{\zeta/(\zeta-1)}(z) = \kappa z^{-\lambda}$$

satisfying **A2** and the strong form of **A3** for $\kappa > 0$ and $\lambda > 0$.

CES for $\kappa = 0$; $\bar{z} = \infty$; $c = 1 - \frac{1}{\sigma}$; CoPaTh for $\bar{z} < \infty$; $c = 1$; $\kappa = \frac{1-\rho}{\rho} > 0$, and $\lambda \to 0$**.**

- $\rho\left(\frac{\psi}{A}\right) = \frac{1}{1+\kappa(z_\psi)^{-\lambda}}$, with $z_\psi = Z\left(\frac{\psi}{A}\right)$ given implicitly by $c\exp\left[\frac{\kappa \bar{z}^{-\lambda}}{\lambda}\right] z_\psi \exp\left[-\frac{\kappa(z_\psi)^{-\lambda}}{\lambda}\right] \equiv \frac{\psi}{A}$,
- $\frac{\partial^2 \ln \rho(\psi/A)}{\partial A \partial \psi} \lesseqqgtr 0 \Leftrightarrow (\kappa)^{\frac{1}{\lambda}} \gtreqqless z_\psi = Z\left(\frac{\psi}{A}\right) \Leftrightarrow \frac{\psi}{A} \lesseqqgtr (\kappa)^{\frac{1}{\lambda}} c\exp\left[\frac{\kappa \bar{z}^{-\lambda} - 1}{\lambda}\right]$; Log-sub(super)modular among more (less) efficient firms. In particular, if $\bar{z} < (\kappa)^{\frac{1}{\lambda}}$, $\frac{\partial^2 \ln \rho(\psi/A)}{\partial A \partial \psi} < 0$ for all $\psi/A < Z(\psi/A) < \bar{z} < \infty$.
- Employment hump-shaped with the peak at $\hat{z} = Z\left(\frac{\hat{\psi}}{A}\right) < \bar{z}$, given implicitly by

$$c\left(1 + \frac{\hat{z}^{\lambda}}{\kappa}\right)\exp\left[-\frac{\kappa \hat{z}^{-\lambda}}{\lambda}\right]\exp\left[\frac{\kappa \bar{z}^{-\lambda}}{\lambda}\right] = 1 \Leftrightarrow \left(1 + \frac{\hat{z}^{\lambda}}{\kappa}\right)\hat{z} = \frac{\hat{\psi}}{A}.$$